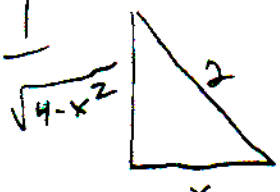
I



$A = \frac{1}{2}bh$

$A(x) = \left(\frac{1}{2}x\right)\sqrt{4-x^2}$ (u, v)

$A'(x) = \frac{1}{2}\sqrt{4-x^2} + \frac{1}{2}x\frac{-x}{\sqrt{4-x^2}}$

$= \frac{1}{2}\left(\frac{4-x^2-x^2}{\sqrt{4-x^2}}\right)$

$= \frac{(4-2x^2)}{2\sqrt{4-x^2}}$

$= \frac{2-x^2}{\sqrt{4-x^2}}$

$= \frac{(\sqrt{2}-x)(\sqrt{2}+x)}{\sqrt{4-x^2}}$

A': + 0 − (0 at $\sqrt{2}$)

max area at $x = \sqrt{2}$ by ~~first~~ first derivative test

$\sqrt{4-2} = \sqrt{2}$

isosceles Rt Δ, $\sqrt{2} : \sqrt{2} : 2$

II (u, v) (u, v) 5pts

$xy + x\sin y = -2\sqrt{x}$

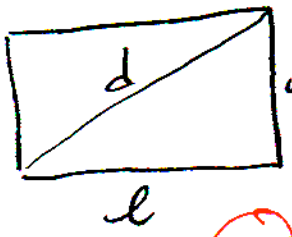
$y + x\frac{dy}{dx} + \sin y + x\cos y\frac{dy}{dx} = -\frac{1}{\sqrt{x}}$

$\frac{dy}{dx} = \frac{-(y+\sin y)}{x + x\cos y}$ 2pts

$= \frac{-2\sqrt{x} + 0}{-1 - 1 \cdot 1} = \frac{-2\sqrt{x}}{-2} = \sqrt{x}$

$-1, 2\sqrt{x}$

III



$\frac{dl}{dt} = 7$

$\frac{dw}{dt} = -3$

(u, v) 5pts

A) $\frac{dA}{dt} = ?$ $A = lw$

$\frac{dA}{dt} = w\frac{dl}{dt} + l\frac{dw}{dt}$

$= 5 \cdot 7 + 12(-3)$

$= 35 - 36$

$= -1$

area is getting smaller at 1 Sq' per second.

B) $\frac{dP}{dt} = ?$ $P = 2l + 2w$

$\frac{dP}{dt} = 2\frac{dl}{dt} + 2\frac{dw}{dt}$

$\frac{dP}{dt} = 2(7) + 2(-3)$

$= 14 - 6$

$= 8$

Perimeter is growing at 8 ft/sec

C) $\frac{dd}{dt} = ?$ $d^2 = l^2 + w^2$

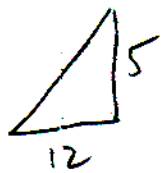
$2d\frac{dd}{dt} = 2l\frac{dl}{dt} + 2w\frac{dw}{dt}$

$\frac{dd}{dt} = \frac{l\frac{dl}{dt} + w\frac{dw}{dt}}{d}$

$= \frac{12 \cdot 7 + 5(-3)}{13}$

$= \frac{84 - 15}{13}$

$= \frac{69}{13}$ 5.308

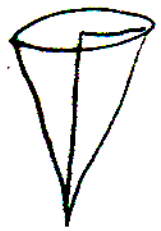


$\frac{144}{25}$

$\frac{169}{} = 13^2$

The diagonal is growing at $\frac{69}{13}$ / per second

IV



$r = \frac{1}{3}h$ $\quad V = \frac{1}{3}\pi r^2 h$

$\frac{dV}{dt} = 10$

$\frac{dh}{dt} = ?$

$h = \frac{30}{\sqrt[3]{\pi}}$

$V = \frac{1}{3}\pi\left(\frac{h^3}{9}\right)$

$V = \frac{\pi}{27}h^3$

$\frac{dV}{dt} = \frac{\pi}{9}h^2\frac{dh}{dt}$

$\frac{90}{\pi}\cdot\frac{1}{h^2} = \frac{dh}{dt}$

$\frac{90}{\pi}\cdot\frac{(\sqrt[3]{\pi})^2}{900} = \frac{dh}{dt}$

$\frac{1}{10\sqrt[3]{\pi}} = \frac{dh}{dt}$ $\quad .068$

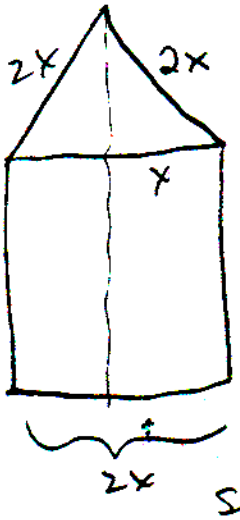
$1000 \overset{\text{set}}{=} \frac{\pi}{27}h^3$

$\frac{27000}{\pi} = h^3$

$\frac{30}{\sqrt[3]{\pi}} = h = 20.484$

The height is rising at the rate of $\frac{1}{10\sqrt[3]{\pi}}$ in per second

V



$P = 6x + 2y \overset{\text{set}}{=} 200$

$3x + y = 100$

$y = 100 - 3x$

$A = lw + \frac{1}{2}bh$

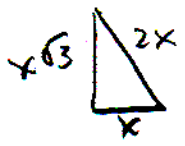
$A(x) = (100-3x)2x + \frac{1}{2}(2x)(x\sqrt{3})$

$A(x) = 200x - 6x^2 + x^2\sqrt{3}$

$A'(x) = 200 - 12x + 2x\sqrt{3} \overset{\text{set}}{=} 0$

$x(6-\sqrt{3}) = 100$

$x = \frac{100}{6-\sqrt{3}}$



1000
160

Base $= 2x$

$\therefore$ Base $= \frac{200}{6-\sqrt{3}}$

$A''(x) = -12 + 2\sqrt{3} < 0$

$\therefore$ max at critical value. f'' test

Extra.

Let w = # of weeks to wait

$10000 - 100w$ > weight

$160 + 10w$ = New Price (cents)

You need to wait 12 weeks to get max income.

$f(x) = (10000 - 100w)(160 + 10w) - 600w$ ← revenue

$100[(100-w)(160+10w) - 6w]$

$100[16000 + 840w - 10w^2 - 600w]$

$f(x) = 100[16000 + 240w - 10w^2]$

$f'(x) = 100[240 - 20w] \overset{\text{set}}{=} 0$

$w = 12$

$f''(x) = 100[-20] < 0$ $\therefore$ max f'' test

change IV so $V \geq 1000\pi^3$ cu" Then we won't have those awful radicals. or Just put "when $h = 10$" That should speed it up.

$\Longrightarrow$

close up the space between x & y in II

	2	5	T
100	0	0	0
90	2	6	8
80	2	6	8
70	4	3	7
60	1	2	3
59	3	3	6

97

8

7510 7625 7518

$\frac{825}{11}$ $\frac{1525}{20}$

5/12/2000

	4	5	T
100	0	0	
90	3	3	
80	5	3	
70	5	5	
60	4	4	
59	4	5	

98

26

$\frac{1488}{20}$ $\frac{1364}{19}$ $\frac{2852}{39}$

74.4 71.8 73.1

Honors Analysis
Copy original problem.
Convince *me* that **you** understand the concept!

Name Analysis ch 4 APP

Per _____ Date 5/22/92

5/12/2000

Chapter 4 Applications Exam

I Among all right triangles with hypotenuse 2, determine (using calculus, of course) the side lengths of the one with the largest area. **Verify**, as usual. (hint: Something *has* to be x.) (15 pts)

II Find $\frac{dy}{dx}$ at $(-1, 2\pi)$ when $xy + x \sin y = -2\pi$ (15 pts)

III The length of a rectangle is increasing at a rate of seven feet per second. The width is decreasing at a rate of three feet per second. When the length is twelve feet and the width is five feet, find the rate of change of: (25 pts tot)

a) the area b) the perimeter c) the length of a diagonal

IV A conical reservoir with radius equal to one-third its height is filling with water at a rate of ten cubic inches per second. How fast is the water level rising when there are one thousand cubic inches of water in the tank? The volume of a cone with height h and radius r is given by: $v = \frac{1}{3}\pi r^2 h$ (20 pts)

V An equilateral triangle is placed on a rectangle as shown: (25 pts)

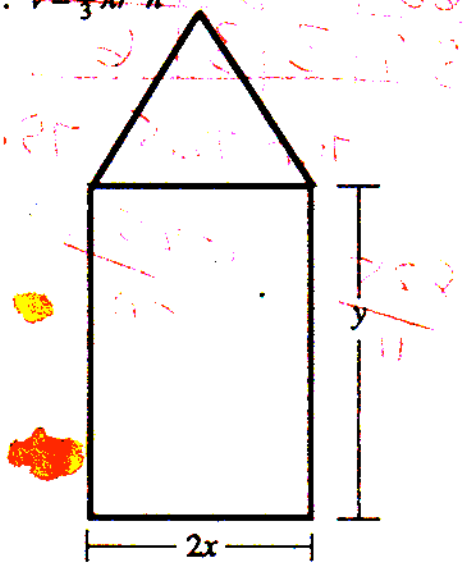
The total perimeter (distance around the exterior) is exactly 200 feet. For the figure to enclose a maximum area, show that the base of the rectangle would be:

$$\frac{200}{6-\sqrt{3}}$$

Justify

Verify using the $f''(x)$ test.

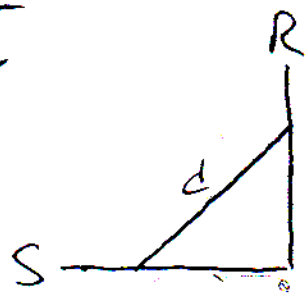
Hint: 30-60-90 triangle —> $1 : 2 : \sqrt{3}$



Extra Credit **5 pts**

Ten thousand pounds of beef in cold storage are worth one dollar and sixty cents per pound, wholesale. If the price increases steadily at ten cents a pound per week while the beef loses a hundred pounds a week in weight (due to shrinkage) and the storage costs are six hundred dollars per week, how many weeks should the beef be held before selling for the greatest net value?

I



2 pts

$\frac{dr}{dt} = -40$

$\frac{dd}{dt} = ?$

$\frac{ds}{dt} = -35$

$d^2 = r^2 + s^2$

$2d\frac{dd}{dt} = 2r\frac{dr}{dt} + 2s\frac{ds}{dt}$

$\frac{dd}{dt} = \frac{r\cdot\frac{dr}{dt} + s\frac{ds}{dt}}{\sqrt{r^2+s^2}}$

$= \frac{-40(9) + 5(-35)}{5}$

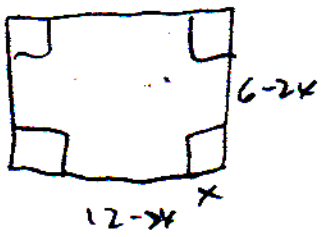
$= -8(9) + 3\cdot(-35)$

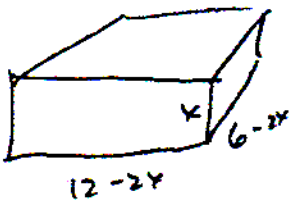
$= -72 - 21$

$\frac{dd}{dt} = -93$

Sandy and Robbie are getting closer at 93 mph.

II





$0 < x < 3$

$V = lwh$

$= (12-2x)(6-2x)(x)$

$V(x) = 72x - 36x^2 + 4x^3$

$V'(x) = 72 - 72x + 12x^2$

$12(x^2 - 6x + 6) \overset{\text{Set}}{=} 0$

$x = \frac{6 \pm \sqrt{36-24}}{2}$

$\frac{6 \pm \sqrt{12}}{2}$

$3 \pm \sqrt{3}$

$2(3+\sqrt{3}) > 6$ ∴ reject. Domain restriction

$V''(x) = 24x - 72$

$V''(3+\sqrt{3}) > 0$ min Reject

$V''(3-\sqrt{3}) < 0$ ∴ max f'' test

the cutout squares should be $(3-\sqrt{3})$ by $(3-\sqrt{3})$

III

$Q(x) = \frac{15}{\sqrt{x}} - \sqrt{x}$

a) $R(x) = xQ(x) = 15\sqrt{x} - x^{\frac{3}{2}}$

$R'(x) = \frac{15}{2\sqrt{x}} - \frac{3\sqrt{x}}{2}$

$C(x) = 3\sqrt{x} + 4$

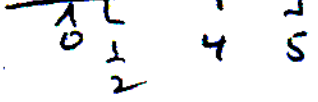
$C' = \frac{3}{2\sqrt{x}}$

$P = 15\sqrt{x} - x^{\frac{3}{2}} - 3\sqrt{x} + 4$

$= 12\sqrt{x} - x^{\frac{3}{2}} + 4$

$P' = \frac{6}{\sqrt{x}} - \frac{3}{2}\sqrt{x}$

b) $\frac{6}{\sqrt{x}} - \frac{3\sqrt{x}}{2} \overset{\text{Set}}{=} 0$



$\frac{12-3x}{2\sqrt{x}} = 0$ the warehouse widget corporation should produce 4,000 widgets to earn maximum profit.

$x = 4$

max f' test

IV

Jake	Ind
Sweet	Sour
$l=2$ $h=4.7$ $w=1$	$r=1$ $h=3$

a) $V = lhw$

$= (2)(4.7)(1)$

$= 9.4 \text{ cm}^3$

$V = \pi(1)^2 3$

3.14

3

9.42

The candies have almost the same volume

b)

Sour $V = \pi r^2 h$ h was constant=3.

$\frac{dV}{dt} = ?$ $V = 3\pi r^2$

$\frac{dr}{dt} = -.1$ $\frac{dV}{dt} = 6\pi r \frac{dr}{dt}$

$(6\pi \frac{1}{2})(\frac{-1}{10})$

$3(3.14)(\frac{-1}{10})$

$\frac{dV}{dt} = -.942$

The Sour Bippy was dissolving at .942 cm^3 per minute.

Sweet

$\frac{dV}{dt} = ?$ height = 4.7 constant.

$\frac{dl}{dt} = \frac{dw}{dt} = \frac{-1}{10}$ $l = 1$

$w = \frac{1}{2}$

$V = (4.7)lw$

$\frac{dV}{dt} = 4.7w\frac{dl}{dt} + 4.7l\frac{dw}{dt}$

$= 4.7\frac{1}{2}\frac{-1}{10} + 4.7\frac{-1}{10}$

$.235$

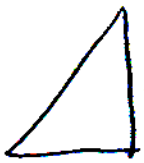
$.470$

$.705$

The Sweet Bippies are melting at .705 cm^3 per min.

So the Sour Bippies melt by .237 cm^3 faster.

Extra



Sandy walk

35 mph

$RT = D$

$T = \frac{D}{R}$

Sandy Rabbit

$T = \frac{3}{35}$ $T = \frac{4}{40} = \frac{1}{10} = \frac{3}{30}$

$\frac{3}{35} < \frac{3}{30}$ ∴ (dividing by a Larger Number)

Sandy requires less time so she arrives first.

.942

.705

237

5/11/2001

	4	5	T	
100	0	1	1	102
90	1	6	7	
80	5	3	8	
70	1	0	1	
60	0	5	5	
59	7	5	12	

1444

20

896 ~~2044~~

14 20

2340

64.0 72.4 68.8

Many chose to sit and chat & think they are still trying to navigate instead of learn.

There were 8 gone for track and for field trip None of the 8 will be stars, maybe some calculus segments will change to stats.

Honors Analysis
Copy original problem.
Convince *me* that **you** understand the concept!
No calculators.
Remember: All problems require meaningful, complete sentences which contain no pronouns.

Name ANL ch 4 APP

Per _____ Date 2001

Chapter 4 Applications Exam

I Sandy and Robbie plan to meet at a restaurant. To get to the restaurant, Sandy is West of the restaurant and is driving East at 35 mph. Robbie is North of the restaurant and is driving South at 40 mph. At what rate is the straight-line distance between Sandy and Robbie changing when Sandy is still 3 miles from the restaurant and Robbie is still 4 miles from the restaurant? (25 pts)

II The Czarina was making fudge to send to her many admirers. She plans to make her own personalized boxes by taking a rectangular piece of cardboard covered with foil that measures twelve by six inches. She will cut out congruent squares out of each corner; folding up all 4 sides of the remaining cardboard and taping where the edges touch. She wants to be as generous as possible so she wants the boxes capable of holding the most fudge. What are the dimensions of the cut out squares? Verify using the $f''(x)$ test. (25 pts)

III In manufacturing and selling the ever popular Wonderous Widget. The Wonderous Widget Corporation found that its price ***per item*** in dollars and cost function are given by: $Q(x) = \frac{15}{\sqrt{x}} - \sqrt{x}$ and $C(x) = 3\sqrt{x} + 4$, respectively. Here x is measured in thousands of widgets and $R(x), C(x)$, and $P(x)$ are measured in hundreds of dollars. Consider values of x such that $\frac{1}{2} \le x \le 5$. (25 pts tot)

a) Find expressions for the revenue, marginal revenue, marginal cost and the marginal profit.

b) Determine the production level that will produce the maximum total profit. (Verify, of course.)

IV The Better Bippy Company offers the original *sweet bippie* which has the shape of a rectangular prism (a rectangular box -- that is the volume = $l\,w\,h$) and the new *sour bippie* in the shape of a cylinder ($V = \pi r^2 h$). Jake likes the sweet bippies which have length 2 cm, height 4.7 cm and width 1 cm. Jed likes the sour bippies, the cylinders have a radius of 1 cm and a height of 3 cm. (Let π=3.14) (25 pts tot)

a) Compare the volume of the two candies.

b) Jed thinks the sour bippies melt faster than the sweet ones. The boys each tried a candy at the same time. They gathered the following information: After 5 minutes, the boys noticed that the heights of the candies remained constant. Furthermore, the cylinder had a the radius of .5, the radius was decreasing at .1 cm/min and for the prism the length was 1 cm and the width was .5 cm. Both the length and width were decreasing at .1 cm per minute. Whose candy was melting faster and by how much?

Extra Credit **5 pts**

Refer to section I above. Who arrives at the restaurant first? Be sure your method is *very clear.*

I

$D(x) = \frac{90x^2}{x^2+48}$ = Percent of pop with disease.

A) Rate of change fastest

$$D'(x) = \frac{(x^2+48)(180x) - 90x^2(2x)}{x^2+48}$$

$$\frac{90x[2x^2+96-2x^2]}{(x^2+48)^2}$$

This is the function we want to see when it is fastest.

$$D'(x) = \frac{96 \cdot 90x}{(x^2+48)^2}$$

$$D''(x) = \frac{(x^2+48)^2 90 \cdot 96 - 90 \cdot 96x \cdot 2(x^2+48)2x}{(x^2+48)^4}$$

B)

$$D'(4) = \frac{96 \cdot 90 \cdot 4}{(16+48)^2} = \frac{90 \cdot 96 (x^2+48)[x^2+48-4x^2]}{(x^2+48)^4}$$

$$= \frac{96 \cdot 90 \cdot 4}{64 \cdot 64} = 8.438 = \frac{90 \cdot 96 (x^2+48)[48-3x^2]}{(x^2+48)^3} \overset{\text{set}}{=} 0$$

The disease will be spreading at about 8% per week.

$3x^2 = 48$

$x^2 = 16$

$x = 4$

D'' f' : + 0 − (0, 4)

max at $x = 4$ weeks f'cal. The disease spreads fastest 4 weeks after it starts

c) $D(4) = \frac{90 \cdot 16}{64} = \frac{45}{2} = 22\frac{1}{2}$

about $22\frac{1}{2}$% of the population will be infected in week 4.

D) $\lim_{x \to \infty} \frac{90x^2}{x^2+48} = 90$

about 90% of the population will get the disease.

II

$S(x) = 8x^3\sqrt{r^2-x^2}$

$S' = \sqrt{r^2-x^2}\,24x^2 - \frac{8x^3(-2x)}{2\sqrt{r^2-x^2}}$

$S'' = \frac{(r^2-x^2)24x^2 - 8x^4}{\sqrt{r^2-x^2}}$

NO K - 15

$\frac{24r^2x^2 - 24x^4 - 8x^4}{\sqrt{r^2-x^2}}$

$\frac{24r^2x^2 - 32x^4}{\sqrt{r^2-x^2}} \overset{\text{set}}{=} 0$

$24r^2 = 32x^2$

$\frac{3r^2}{4} = x^2$

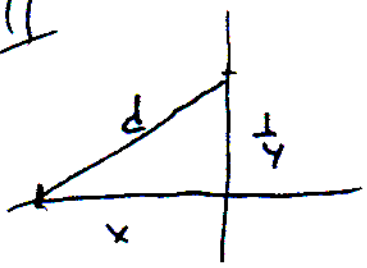
$\frac{r\sqrt{3}}{2} = x$

S' : + 0 − (0, $\frac{r\sqrt{3}}{2}$)

max at $\frac{r\sqrt{3}}{2}$ f'cal. The beam will be $r\sqrt{3}$ high and $\frac{r}{2}$ wide

$\sqrt{r^2 - \frac{3r^2}{4}} = \frac{r^2}{4}$

11



$\frac{dd}{dt} = -30$

$\frac{dx}{dt} = -50$

$x = ?$

$x^2 + \frac{1}{16} = d^2$

$2x\frac{dx}{dt} = 2d\frac{dd}{dt}$

$-50x = -30d$

$x = \frac{3}{5}d$

$d = \frac{\sqrt{16x^2+1}}{4}$

$x = \frac{3}{5}\cdot\frac{\sqrt{16x^2+1}}{4}$

$20x = 3\sqrt{16x^2+1}$

$400x^2 = 9(16x^2+1)$

$= 144x^2 + 9$

$256x^2 = 9$

$x^2 = \frac{9}{256}$

$x = \frac{3}{16}$

The sparks can will be $\frac{3}{16}$ of a mile from the intersection

10

$R = xq(x) = x^3 - 195x^2 + 24000x$

$R' = 3x^2 - 390x + 24000$

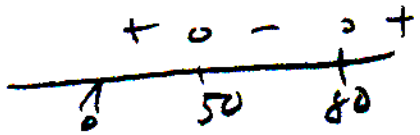
$P = R - C = x^3 - 195x^2 + 24000x - (12000x + 200000)$

$= x^3 - 195x^2 + 12000x - 200000$

$P' = 3x^2 - 390x + 12000$

$3(x^2 - 130x + 4000)$

$3(x - 50)(x - 80)$ set $= 0$



max at $x = 50$ feet.

The company should sell 50 units for max Profit.

$3x^2 - 390x + 12000$

$3(x^2 - 130x + 4000)$

5/17/2001

Jehnie Cutts	20
Brittany Gonzales	60
Wanking Lewis	45
Pascha McAllister	50
Sunday Sankaran	45
Homa Karimi	45
Brandon Reyes	45
Becki Neal	zero

No surprises. They weren't prepared for the regular exam and (probably) expected last year's make up exam. It didn't happen. They got what they earned.

Honors Analysis Name AWL Ch 4 APP

Copy original problem. Per _____ Date _____

Convince *me* that **you** understand the concept!

Each problem is worth 25 pts.

No calculators

Chapter 4 Applications Make Up Exam

I A communicable disease has just been discovered in a particular community. Typically during the first x weeks, $D(x) = \dfrac{90x^2}{x^2+48}$ percent of the population contract the disease. (30 pts tot)

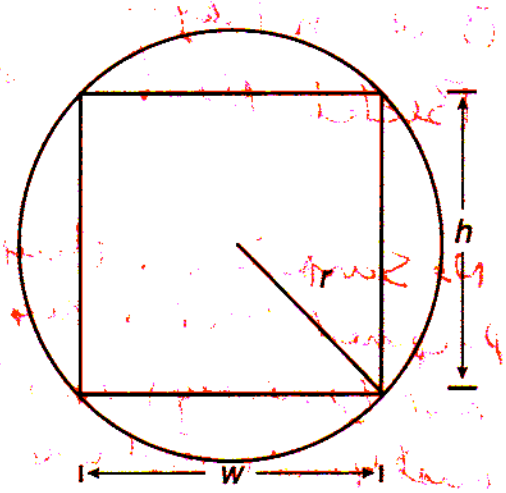
a) When will the disease be spreading the fastest?

b) How fast will the disease be spreading at the time it is spreading fastest?

c) What percentage of the population will have been infected at the time the disease is spreading its fastest?

d) What percent of the population will ultimately become infected?

II The stiffness of a rectangular beam is equal to the product of the width and the cube of the height of the beam. Consider the figure. In terms of r, the radius of the log (a right circular cylinder) from which the beam will be cut, what are the dimensions of the beam with maximum stiffness?



Reread

III Maple and Main Streets are straight and perpendicular to each other. A stationary police car is located on Main Street, one quarter mile from the intersection. A sports car on Maple Street approaches the intersection in such a way that the distance between the sports car and the police car decreases at 30 mph. How far from the intersection would the sports car be at the moment the sports car is traveling at 50 mph?

Problem wording. Sports car is always traveling at 50 mph ($-50 = \frac{dy}{dt}$)

IV A certain business had some studies done and found the relationship of $q(x)$, the price ***per unit***, and x, the number of units which would be sold was $q(x) = x^2 - 195x + 24000$. The total production cost is given by $c(x) = 12000x + 200000$. Due to size limitations, the business could produce sixty items a month. The real question was, could the company make more profit if it produced less than sixty items per month?

a) Identify and state the revenue function, marginal revenue function, profit function and marginal profit function.

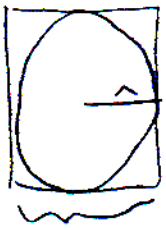
b) What should the production level be so the company will have maximum profit?

I

A) $\frac{dc}{dt} = 6$ $\quad \frac{dr}{dt} = 6$

$P = 4R$ $\quad -5$

$\frac{dP}{dt} = ?$



B) $A = 4r^2 - \pi r^2$ $\quad C = 2\pi r$ $\quad -6$

$\frac{dA}{dt} = ?$ $\quad \frac{dC}{dt} = 2\pi \frac{dr}{dt}$

$\frac{dA}{dt} = 8r\frac{dr}{dt} - 2\pi r\frac{dr}{dt}$ $\quad \frac{6}{2\pi} = \frac{dr}{dt}$

$\frac{3}{\pi} = \frac{dr}{dt}$ $\quad = 3\pi$ $\quad -4$

$P = 8r$ $\quad C = 2\pi r$ $\quad$ or $\quad P = 8r$ $\quad C = \pi r^2 = 25\pi$ $\quad r = 5$

$P = 8\left(\frac{C}{2\pi}\right) = \frac{4C}{\pi}$

$\frac{dP}{dt} = \frac{4}{\pi}\frac{dC}{dt}$ $\quad \frac{dP}{dt} = 8\frac{dr}{dt}$

$= \frac{4}{\pi} \cdot 6$ $\quad \frac{dP}{dt} = 8 \cdot \frac{3}{\pi}$

$\frac{dP}{dt} = \frac{24}{\pi}$ $\quad = \frac{24}{\pi}$

$\frac{dA}{dt} = 8 \cdot 5 \cdot \frac{3}{\pi} - 2\pi 5 \cdot \frac{3}{\pi}$

$= \frac{120}{\pi} - 30$

$\frac{dA}{dt} = \frac{30}{\pi}(4 - \pi) = 8.197$

the Perimeter is expanding at $\frac{24}{\pi}$ u/sec

the area is expanding at $\frac{30}{\pi}(4-\pi)$ sq "/sec.

7.639

II

$P = 2r + r\theta$ $\quad \frac{1}{2}r^2\theta = 4\pi$ $\quad r^2 = \frac{8\pi}{\theta}$ $\quad r = \sqrt{\frac{8\pi}{\theta}}$

$P(\theta) = 4\sqrt{\frac{2\pi}{\theta}} + 2r\sqrt{\frac{2\pi}{\theta}} \cdot \theta$

$= 4\sqrt{\frac{2\pi}{\theta}} + 2\sqrt{2\pi\theta}$

$= 4\sqrt{2\pi}\theta^{-\frac{1}{2}} + 2\sqrt{2\pi}\theta^{\frac{1}{2}}$ $\quad \pi = 3.645$ $\quad \theta = 2$

$P' = \frac{4\sqrt{2\pi}}{-2\theta^{\frac{3}{2}}} + \frac{\sqrt{2\pi}}{\theta^{\frac{1}{2}}} \overset{set}{=} 0$

$-\frac{2\sqrt{2\pi}}{\theta^{\frac{3}{2}}} + \frac{\sqrt{2\pi}}{\theta^{\frac{1}{2}}} = 0$

$\frac{2}{\theta^{\frac{3}{2}}} = \frac{1}{\theta^{\frac{1}{2}}}$

$2 = \theta$

$P' = -2\sqrt{2\pi}\theta^{-\frac{3}{2}} + \sqrt{2\pi}\theta^{-\frac{1}{2}}$

$P'' = +3\sqrt{2\pi}\theta^{-\frac{5}{2}} - \frac{\sqrt{2\pi}}{2}\theta^{-\frac{3}{2}} = \frac{\sqrt{2\pi}}{\theta^{\frac{3}{2}}}\left(\frac{3}{\theta} - 1\right)$

$P''(2) > 0$ $\therefore$ min at $\theta = 2$. f "cst

$P'' = \frac{\sqrt{2\pi}}{\theta^{\frac{3}{2}}}\left(\frac{3}{2} - 1\right) > 0$

θ should be radians.

OR

$P = 2r + r\theta$

$\theta = \frac{8\pi}{r^2}$

$P = 2r + r \cdot \frac{8\pi}{r^2}$

$P(r) = 2r + \frac{8\pi}{r}$

$P' = 2 - \frac{8\pi}{r^2} \overset{set}{=} 0$

$\frac{8\pi}{r^2} = 2$

$4\pi = r^2$

$r = 2\sqrt{\pi}$

$P'' = \frac{16\pi}{r^3} > 0$ $\therefore$ min.

$r = \sqrt{\frac{8\pi}{\theta}}$

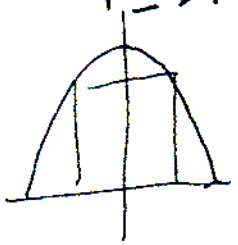
$2\sqrt{\pi} = \sqrt{\frac{2\pi}{\theta}}$

$\sqrt{\theta} = \sqrt{2}$

$\theta = 2$

III

4)



(3, 16)

base = 6
height = 16

Area 96
Perimeter = 12+32
= 44

B)

$A = 2x(25 - x^2)$
$= 50x - 2x^3$
$A' = 50 - 6x^2 \overset{\text{set}}{=} 0$
$x^2 = \frac{25}{3}$
$x = \frac{5\sqrt{3}}{3} \approx 2.887$

$A''(x) = -12x$
$A''\left(\frac{5\sqrt{3}}{3}\right) < 0$ ∴ max
f "cc",

Area $= 2 \cdot \frac{5\sqrt{3}}{3} \cdot \frac{50}{3}$
$\frac{500\sqrt{3}}{9}$ sq inches $= 96.225$

Perimeter $= \frac{20\sqrt{3}}{3} + \frac{100}{3}$
$\frac{20}{3}(\sqrt{3} + 5)$ inches
44.880

C) $P = 4x + 2y$
$P = 4x + 2(25 - x^2)$
$= 4x + 50 - 2x^2$
$P' = 4 - 4x \overset{\text{set}}{=} 0$
$x = 1$

$P'' = -4$
max f "cc".

Perimeter $4 \cdot 1 + 48$
52

area $2x(25 - (1))$
$2(24)$
48

d) No. The answers are not similar and I didn't expect them to be.

Extra x = Prey
y = Predator

$y = \frac{1}{6}x^2 + 90$

$x = 7t + 85$

$\frac{dy}{dt}\Big| = ?$ $\quad \frac{dx}{dt} = 7$

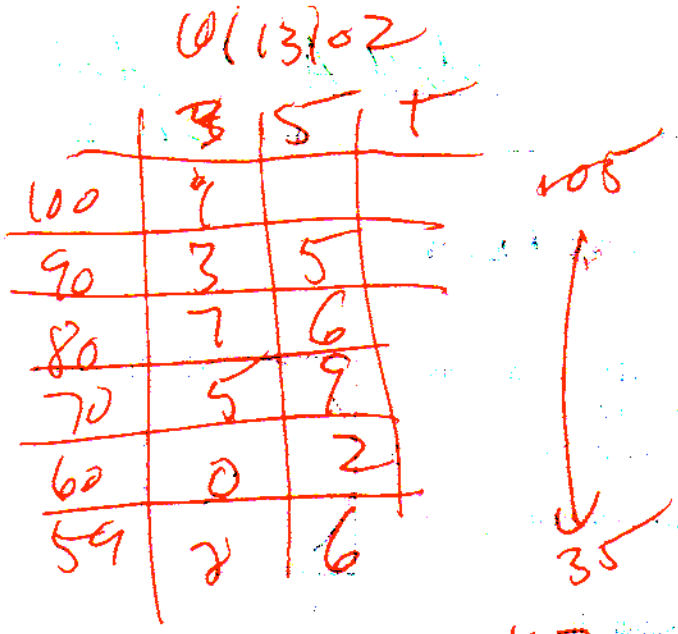
$t = 8$ $\quad \frac{dy}{dt} = \frac{1}{3} \times \frac{dx}{dt}$

$t = 8 \rightarrow x = 56 + 85$
$= 141$

$\frac{dy}{dt} = \frac{1}{3} \cdot 141 \cdot 7$

$= 329$

The population of Predators is growing at 329 Predators per week.



1431 / 18 = 79.5

2034 / 28 = 72.6

3465 / 46 = 75.3

Amazing!

This was a shortening of 1994. Interpreted 85+ average.

Simply amazing.

(w/ underlapping with phlm 3

94

	2	5	T
100	0	0	0
90	0	2	2
80	3	5	8
70	3	2	~~5~~
60	6	0	6
59	8	12	20

94 ↓ 0

1045 1133
~~20~~ 21
14
~~5~~ 55,0 53.95

2178 ~~5~~ 54.45
40

Students "expected" to get back points on the ~~test~~ "like last year..." they didn't.

5/16/97

100	0
90	2
80	7
70	1
60	7
54	7

94 ↓ 35

68.4

Honors Analysis Name Ch 4 APP ANL

Copy original problem. Per _____ Date 2002

Convince *me* that you understand the concept!

Chapter 4 Applications Exam

You may use a calculator on this exam.

I

A circle is inscribed in a square as shown in the figure. The circumference of the circle is increasing at a constant rate of six inches per second. As the circle expands, the square expands to maintain the condition of tangency. (40 pts tot)

a) Find the rate at which the **perimeter** of the ***square*** is increasing. Indicate units of measure.

b) At the instant *when* the area of the circle is 25π square inches, find the rate of increase in the area **enclosed between** the circle and the square. Indicate units of measure.

$A_{circle} = \pi r^2$

$C_{circle} = 2\pi r$

II

A region is in the shape of a wedge of a circle as shown. If the total area of the wedge is 4π, what should θ be to minimize the perimeter? Verify using f'' test. (30 pts)

$Area_{sector} = \frac{1}{2} r^2 \theta$

$Perimeter_{sector} = 2r + r\theta$

III

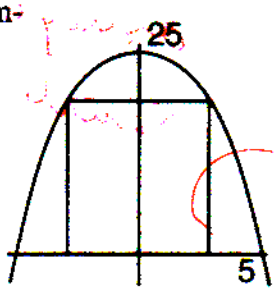
A rectangle has two of its vertexes on the x - axis and the other two above the x - axis and on the graph of the parabola: $y = 25 - x^2$. See the picture on the right. (30 pts tot)

a) If the coordinates of **one** of the vertexes is $(3, 16)$, what is the area and perimeter of the rectangle?

b) What is the area and perimeter of the rectangle which has *largest* **area** ?

c) What is the area and perimeter of the rectangle which has *largest* **perimeter** ?

d) Are you surprised by the similarity of the answers of parts b and c?



Extra Credit ---------- 5 pts ----------

In a forest a predator feeds on prey, and the predator population at any time is a function of the number of prey in the forest at that time. Suppose that when there are x prey in the forest, the predator population is y and $y = \frac{1}{6}x^2 + 90$. Furthermore, if t weeks have elapsed since the end of the hunting season, $x = 7t + 85$. At what rate is the population of the predator growing 8 weeks after the close of the hunting season? Do not express y in terms of t, but use the chain rule.

I

a) $D(x) = \frac{90x^2}{x^2+48}$ u

5 pts each + 5 pts for unity.

$D'(x) = \frac{(x^2+48)180x - 90x^2 \cdot 2x}{(x^2+48)^2}$

$= \frac{180x(x^2+48-x^2)}{(x^2+48)^2}$

$= \frac{180x \cdot 48}{(x^2+48)^2}$ u

$D''(x) = \frac{(x^2+48)^2 \cdot 180 \cdot 48 - 180x \cdot 48 \cdot 2 \cdot 2x(x^2+48)}{(x^2+48)^4}$

$= \frac{180 \cdot 48[x^2+48-4x^2]}{(x^2+48)^3} \le 0$

$3x^2 = 48$ $\quad D''$: + 0 −

$x = 4$ $\quad$ max at $x=4$ f'test.

b) $D'(4) = \frac{180 \cdot 4 \cdot 48}{(16+48)^2} = 8.4375$

disease will be spreading at about 8% per week at that time.

c) $D(4) = \frac{90 \cdot 16}{16+48} = 22\frac{1}{2}\%$

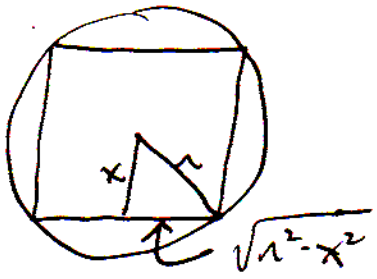
about 22.5% of the population will be infected.

d) $\lim_{x\to\infty} \frac{90x^2}{x^2+48} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$

$\lim_{x\to\infty} \frac{90}{1+\frac{48}{x^2}} = 90$

ultimately 90% of the population will get the disease.

II



height $= 2x$

$f(x) = 8x^3 \cdot 2\sqrt{r^2-x^2}$

$= 16x^3\sqrt{r^2-x^2}$

$f'(x) = \sqrt{r^2-x^2} \cdot 48x^2 + kx^3 \cdot \frac{-2x}{2\sqrt{r^2-x^2}}$

$= \frac{(r^2-x^2)48x^2 - 16x^4}{\sqrt{r^2-x^2}}$

$= \frac{16x^2[(r^2-x^2)\cdot 3 - x^2]}{\sqrt{r^2-x^2}}$

$= \frac{16x^2[3r^2-4x^2]}{\sqrt{r^2-x^2}} \le 0$ set

$3r^2 = 4x^2$

$x^2 = \frac{3}{4}r^2$

$x = \frac{r}{2}\sqrt{3}$

The Beam will be $r\sqrt{3}$ wide and the height is r

$2\sqrt{r^2-\frac{3}{4}r^2}$

$2\sqrt{\frac{r^2}{4}} =$

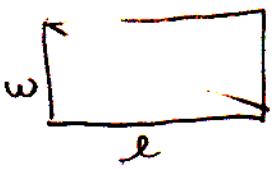
$2\frac{r}{2} = 1$

f': + 0 −

0 $\quad \frac{r\sqrt{3}}{2}$

max f'test.

III

a) 

$\frac{dl}{dt} = 7$ $\frac{dA}{dt} = ?$

$\frac{dw}{dt} = -3$ $l = 12$

$w = 5$

$A = lw$ No product rule -10

$\frac{dA}{dt} = w\cdot\frac{dl}{dt} + l\cdot\frac{dw}{dt}$

$= 5\cdot 7 + 12(-3)$

$= 35 - 36$

$= -1$

area is getting smaller at 1 ft²/sec.

b)

$\frac{dP}{dt} = ?$

$P = 2w + 2l$

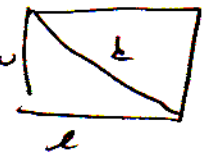
$\frac{dP}{dt} = 2\frac{dw}{dt} + 2\frac{dl}{dt}$

$= 2(-3) + 2(7)$

$= -6 + 14$

$= 8$

The perimeter is increasing at 8 ft/sec.

c) 

$\frac{dd}{dt} = ?$ $d^2 = w^2 + l^2$

$2d\frac{dd}{dt} = 2w\frac{dw}{dt} + 2l\frac{dl}{dt}$

$d\cdot\frac{dd}{dt} = w\frac{dw}{dt} + l\frac{dl}{dt}$

$= \frac{5(-3) + 12(7)}{13}$

$\frac{dd}{dt} = \frac{-15 + 84}{13}$

$= \frac{69}{13} \approx 5.308$

The diagonal is increasing at $\frac{69}{13}$ ft/sec.

IV

Revenue $R(x) = xp = x^3 - 195x^2 + 24000x$

$R'(x) = 3x^2 - 390x + 24000$

$P(x) = R(x) - C(x) = x^3 - 195x^2 + 24000x - (12000x + 200000)$

$= x^3 - 195x + 12000x - 200000$

$P'(x) = 3x^2 - 390x + 12000 \overset{\text{Set}}{=} 0$

$3(x^2 - 130x + 4000) = 0$

$3(x - 50)(x - 80) = 0$

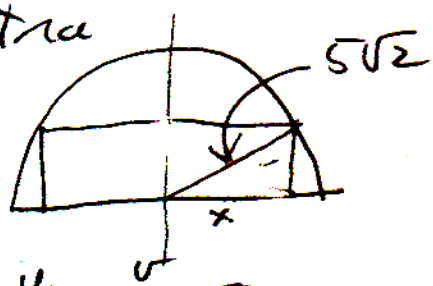
$x \in \{50, 80\}$

	0		50		80	
	+	0	−	0	+	

max at producing 50 units

$P'(x)$.

Extra



u v

$A(x) = 2x\sqrt{50 - x^2}$

$A'(x) = 2\sqrt{50 - x^2} + \frac{2x(-2x)}{2\sqrt{50 - x^2}}$

$\frac{2(50 - x^2) - 2x^2}{\sqrt{50 - x^2}}$

$= \frac{100 - 4x^2}{\sqrt{50 - x^2}} \overset{\text{Set}}{=} 0$

$x^2 = 25$

$x = 5$

	0		5	
	+	0	−	

2 pts

max $P'(x)$.

The dimensions of the rectangle are 5 × 10

100	0	0	0
90	3	0	3
80	2	2	4
70	3	4	7
60	1	5	6
59	7	9	16

97

17

$\frac{946}{14}$ $\frac{1196}{19}$

67.6 62.9 64.5

Honors Analysis

Copy original problem.

Convince *me* that **you** understand the concept!

Each section is worth 25 pts.

Name AnL Ch4 App

Per _____ Date 5/23/03

Chapter 4 Applications Exam

I A communicable disease has just been discovered in a particular community. Typically during the first x weeks, $D(x) = \dfrac{90x^2}{x^2 + 48}$ percent of the population will contract the disease.

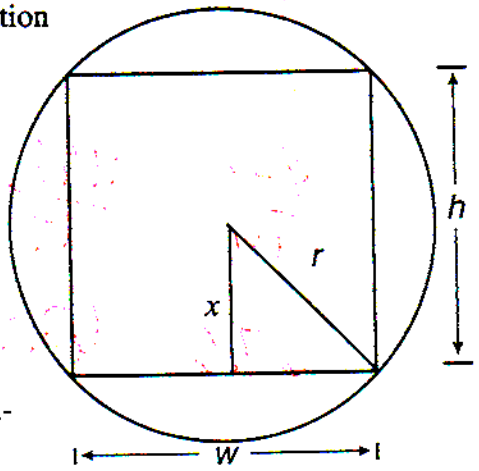
a) When will the disease be spreading the fastest? (hint: When will the rate of change of D be the fastest? ie. $D''(x) \overset{set}{=} 0$)

b) To the nearest percent, how fast will the disease be spreading at the time it is spreading fastest?

c) To the nearest percent, what percentage of the population will have been infected at the time the disease is spreading its fastest?

d) What percent of the population will ultimately become infected? (Does this sound like a limit problem?)

II The stiffness of a rectangular beam is equal to the product of the width and the cube of the height of the beam. Consider the figure. In terms of r, the radius of the log (a right circular cylinder) from which the beam will be cut, what are the dimensions of the beam with maximum stiffness? (x is the variable and r is a constant!)



III The length of a rectangle is increasing at a rate of seven feet per second. The width is decreasing at a rate of three feet per second. When the length is twelve feet and the width is five feet, find the rate of change of: (Draw a new picture for each part.)

a) the area b) the perimeter c) the length of a diagonal

IV A certain business had some studies done and found the relationship of $q(x)$, the price ***per unit***, and x, the number of units which would be sold was $q(x) = x^2 - 195x + 24000$. The total production cost is given by $c(x) = 12000x + 200000$. Due to size limitations, the business could produce sixty items a month. The real question was, could the company make more profit if it produced less than sixty items per month?

a) Identify and state the revenue function, marginal revenue function, profit function and marginal profit function.

b) What should the production level be so the company will have maximum profit?

Extra Credit ---------- 5 pts ----------

Find the dimensions of the rectangle with greatest area which can be inscribed in a **semi**-circle of radius $5\sqrt{2}$.

Honors Analysis

Chapter 4 Applications

2003

Exam Chapter 4 Mech

1 R/C Pg 158 - 163

2 Explain in **great detail** the origin of **each** of the equations listed in the oral exercises on Pg 163

4 / 25

3 Pg 163 1 - 6

4 Pg 164 7 - 15

4/ 28

5 Pg 164 16 - 21
Do NOT use method of examples.
Use chain rule

6 Pg 184 17, 18

4 / 29

7 Pg 184 19 , 20

8 Pg 188 7, 9, 10
do pblm 19 using x as variable instead of r
Important Lecture

4 / 30

9 Pg 189 11, 12

10 "Applying the Derivative" part I ONLY

5 / 1

11 "A the D" 2 – 4

5 / 2

12 "The Hamburger Stand"

Important Lecture

5 / 5

13 "A the D" 5 - 7

Use your grapher to solve the cubic equation in problem 7

5 / 6

14 "A the D" 8 - 10

5/7

15 "A the D" 11 - 13

5/8

16 "A the D" 14 - 16

5/9

You must include a brief explanation of your method for each problem

19 "A the D" 17 - 19

5/12

20 "A the D" 20 - 22

5/ 13

21 "A the D" 23 - 24

5/14

You must include a brief explanation of your method for each problem

Put Last Year's Exam on the Chalkboards

5/15

Exam Ch 4 *Applications* 2003

5/16

I

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = -15$

$\frac{dx}{dt} = ?$

$r = 10 + x$

$V = \frac{4}{3}\pi(10+x)^3$

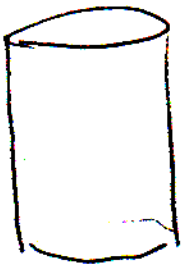
$\frac{dV}{dt} = 4\pi(10+x)^2 \frac{dx}{dt}$

$-15 = 4\pi(15)^2 \frac{dx}{dt}$

$-\frac{1}{60\pi} = \frac{dx}{dt}$

the ice is decreasing at $\frac{1}{60\pi}$ " per minute

II



$V = \pi r^2 h = 27\pi$

$h = \frac{27}{r^2}$

surface area

$S = \pi r^2 + 2\pi r h$

$S(r) = \pi r^2 + 2\pi r \cdot \frac{27}{r^2}$

$S(r) = \pi r^2 + \frac{54\pi}{r}$

$S'(r) = 2\pi r - \frac{54\pi}{r^2} \overset{\text{set}}{=} 0$

Verify Spts

$r^3 = 27$

$r = 3$

$h = \frac{27}{r^2}\Big| = 3$

$r = 3$

Can is 3" high & has radius of 3.

$S''(r) = 2\pi + \frac{108\pi}{r^3} > 0 \ \forall r$, $\therefore$ min f'' test

Do not use product rule -10

III

$V = \pi r^2 h$

$\frac{dr}{dt} = -1$

$\frac{dh}{dt} = 4$

$\frac{dV}{dt} = ?$

$V = \pi r^2 h$

$12\pi = \pi \cdot 4 h$

$h = 3$

$\frac{dV}{dt} = 2\pi r h \frac{dr}{dt} + \pi r^2 \frac{dh}{dt}$

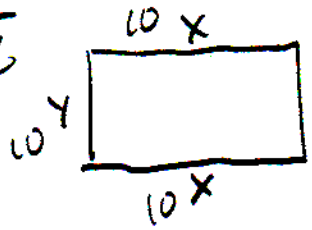
$\frac{dV}{dt} = 2\pi(3)(3)(-1) + \pi(4)(4)$

$= -12\pi + 16\pi$

$\frac{dV}{dt} = 4\pi$

volume getting larger at 4π cu"/hr.

IV



$xy = 1000$

$C = 10(2x) + 10y + 8y$

$= 20x + 18y$

$C(x) = 20x + \frac{18000}{x}$

$C'(x) = 20 - \frac{18000}{x^2} \overset{\text{set}}{=} 0$

$20x^2 = 18000$

$x^2 = 900$

$x = 30$

$y = \frac{1000}{30}$

$= \frac{100}{3}$

$C(30) =$

$600 + \frac{18000}{30}$ (600 + 18000/30)

$C(30) = \$1200$

$C'' = \frac{36000}{x^3} < 0$

$\therefore$ min f'' test.

V. let x = 1st # $f(x) = x^3 + (\frac{1}{2}-x)^2$

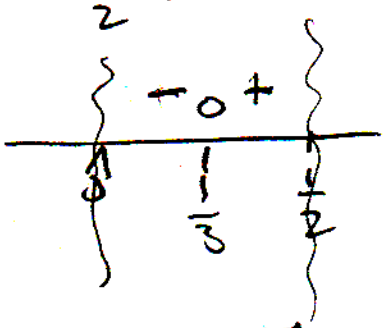
$\frac{1}{2}-x$ = 2nd # $f'(x) = 3x^2 + 2(\frac{1}{2}-x)(-1) \overset{\text{set}}{=} 0$

$3x^2 - 1 + 2x = 0$

$3x^2 + 2x - 1 = 0$

$(3x-1)(x+1)$

$x = \frac{1}{3}$ $x = -1$ reject.



min at $x = \frac{1}{3}$ f'cost.

A) max at ENDpts. "decrease from zero"

∴ 0, $\frac{1}{2}$ are the 2 #s

B) $\frac{1}{3}, \frac{1}{6}$ yield min. f'cost.

VI $C(x) = 100 + \frac{1}{2}x^2 + \sqrt{x}$

A) Av cost $= \frac{C(x)}{x} = \frac{100}{x} + \frac{1}{2}x + \frac{1}{\sqrt{x}}\Big|_{x=100} = 1 + 50 + \frac{1}{10} =$ 51.1

B) Marg Cost $= C'(x) = x + \frac{1}{2}x^{-\frac{1}{2}}\Big|_{x=100} = 100 + \frac{1}{2 \cdot 10} = 100.05$

Extra credit

$\frac{dx}{dt} = \frac{-1}{60\pi}$

$\frac{dS}{dt} = ?$

$S = 4\pi r^2$

$r = 10 + x$

$S = 4\pi(10+x)^2$

$\frac{dS}{dt} = 8\pi(10+x)\frac{dx}{dt}$

$= 8\pi(+5)\left(\frac{-1}{60\pi}\right)$

$\frac{dS}{dt} = -2$

the surface area is decreasing at 2 sq" per minute.

Excellent exam.
No changes

While only 4 or 5
passed the exam,
only 4 or 5 did
any hw. ...
they earned it.
they should
receive it.

ANALYSIS CHAPTER 4 | APPLICATIONS NAME Ch 4 Analysis

Convince ME that YOU do know the concepts !! PER Apple 5/8/87

I A spherical iron ball twenty inches in diameter is coated with a layer of ice of uniform thickness. Assuming that the ice is melting at the rate of 15 CUBIC inches per minute, how fast is the thickness of the ice decreasing when the ice is five inches thick?

(Hint: $V = \frac{4}{3}\pi r^3$ $S = 4\pi r^2$) (10 pts)

II A cylindrical can with an open top is made to contain a fixed volume of 27 π cubic inches. Find the dimensions of the can such that the least amount of material is used. (hint:

($V = \pi r^2 h$ Area of circle = πr^2 Side of cylinder = $2\pi rh$) (20 pts)

III The radius of a right circular cylinder (another can) is decreasing at the rate of 1 inch per hour. The height is increasing at the rate of 4 inches per hour. At the moment when the volume is 12 π cubic inches, the radius of the base is 2 inches. Is the volume increasing or decreasing at that moment? At what rate? (20 pts)

IV A rectangular area of 1000 square yards is to be fenced in. The fencing on three sides is to cost ten dollars per yard. The fourth side calls for a cheaper quality fence costing only eight dollars per yard. Find the least amount that the fencing material can cost. (20 pts)

V Two numbers greater than or equal to zero have a sum of one-half. Find these two numbers if the cube of one of them plus the squareof the other is to be (20 pts tot)

A) a maximum.

B) a minimum.

VI The total cost $C(x) = 100 + \frac{1}{2}x^2 + \sqrt{x}$ of producing x units of a commodity. (10 pts tot)

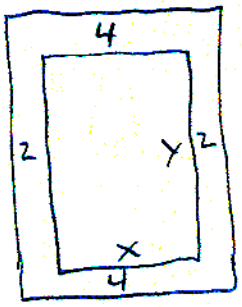
A) Find the average cost for producing 100 units.

B) Find the marginal cost for producing 100 units.

EXTRA CREDIT ——————————— 5 pts

How fast is the outer surface of ice decreasing on the ball in section I when the ice is five inches thick?

I



$xy = 50 \quad y = \frac{50}{x}$

$A = (x+4)(y+8)$

$A(x) = (x+4)\left(\frac{50}{x}+8\right)$

$A'(x) = \left(\frac{50}{x}+8\right) + (x+4)\left(-\frac{50}{x^2}\right)$

$= \frac{50x+8x^2-50x-200}{x^2}$

$= \frac{8x^2-200}{x^2}$

$= 8 - \frac{200}{x^2} \overset{\text{set}}{=} 0$

$8x^2 = 200$

$x^2 = 25$

$x = \pm 5$

$x = 5$

$y = 10$

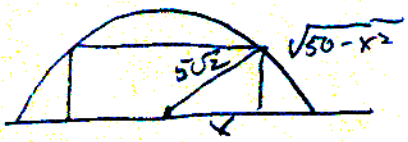
$A'' = \frac{400}{x^3}$

$A'' < 0$, min

Poster is $5+4 \times 10+8$

9×18

II



$A = lw$

$A = 2x\sqrt{50-x^2}$

$A'(x) = 2\sqrt{50-x^2} + \frac{2x(-2x)}{2\sqrt{50-x^2}}$

$A'(x) = \frac{100-2x^2-2x^2}{\sqrt{50-x^2}}$

A': + 0 − (−5, 0, 5)

$\frac{100-4x^2}{\sqrt{50-x^2}} = 0$

max f' $x^2 = 25$

$x = 5$

$h = \sqrt{50-25} = \sqrt{25} = 5$

Dimensions 5×10

III

$\frac{dv}{dt} = 2\pi$

$\frac{dr}{dt} = \frac{1}{2}$

A) $A = \pi r^2 \quad \frac{dA}{dt} = ?$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$= 2\pi(3)\left(\frac{1}{2}\right)$

$\frac{dA}{dt} = 3\pi$

area is changing at 3π sq units per second

B) $\frac{dh}{dt} = ? \quad V = \left(\frac{1}{3}\pi r^2\right)(h)$

3 pt

$\frac{dv}{dt} = h\left(\frac{2}{3}\pi r \frac{dr}{dt}\right) + \frac{1}{3}\pi r^2 \frac{dh}{dt}$

$2\pi = 4\left(\frac{2}{3}\pi(3)\left(\frac{1}{2}\right)\right) + \frac{1}{3}\pi(3)^2\frac{dh}{dt}$

$2\pi = 4\pi + 3\pi\frac{dh}{dt}$

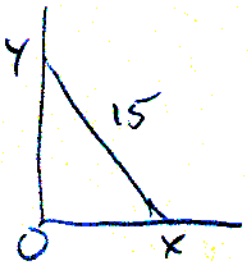
$-2\pi = 3\pi\frac{dh}{dt}$

Product Rule −5

$\frac{dh}{dt} = -2$ The height is decreasing at 2 units per second

$V = \frac{1}{3}\pi r^2 h \quad 12\pi = \frac{1}{3}\pi 3^2 h \quad 4 = h$

10



A) $\left.\frac{dy}{dt}\right|_{x=9} = ?$ $\quad \frac{dx}{dt} = \frac{1}{2}$

$x^2 + y^2 = 15^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$

$= -\frac{9}{12}\left(\frac{1}{2}\right)$

$= -\frac{3}{8}$

Falling at $\frac{3}{8}$ ft/sec.

$81 + y^2 = 225$

$y^2 = 144$

$y = 12$

B) $\left.\frac{dA}{dt}\right|_{x=9}$

$A = \frac{1}{2}xy$

$\frac{dA}{dt} = \frac{1}{2}\left(y\frac{dx}{dt} + x\frac{dy}{dt}\right)$

$\frac{dA}{dt} = \frac{1}{2}12\left(\frac{1}{2}\right) + \frac{1}{2}9\left(-\frac{3}{8}\right)$

$= 3 - \frac{27}{16}$

$\frac{48-27}{16} = \frac{21}{16}$

Area increasing at $\frac{21}{16}$ sq ft/sec.

Extra,

$\frac{dA}{dh} = ?$

$\frac{dA}{dh} = \frac{dA}{dt}\cdot\frac{dt}{dh}$

$= 3\pi \cdot \frac{1}{8}$

$=$ Part A · Part B Answers

$\frac{dA}{dt} = \frac{3\pi}{8}$.

11

A) $C(x) = 100 + \frac{1}{2}x^2 + \sqrt{x}$

$C(100) = 100 + \frac{1}{2}(100)(100) + 10$

$\frac{C(100)}{100} = 1 + 50 + \frac{1}{10} = 51.1$

B) $C'(x) = x + \frac{1}{2\sqrt{x}}$

$C'(100) = 100 + \frac{1}{2(10)}$

$= 100\frac{1}{20}$

$= 100.05$

include at least 2
more days on this unit.
Collect "applying" and grade it.
as 88 then have students fix it
and put on boards.
(calculus will have to put plans on
board during 3rd period.

Maybe on Thurs collecting —
Just put on board for 5 pts ~~each~~.
each — volunteer assignment is worth
5 pts too. extra is extra credit.
Then would require only 1 extra day.
and could use ~~V~~ front boards only.

100	2	2	4
90	4	2	6
80	3	2	5
70	2	2	4
60	3	1	4
59	8	10	18
	68.0	60.0	63.3

104

5

1988

ANALYSIS CHAPTER 4 APPLICATIONS NAME Ch4 App 1988

Convince ME that YOU do know the concepts !! PER _____

I A poster with fifty square inches of printed material is to have top and bottom margins of four inches each and side margins of two inches each. Find the OVERALL dimensions of the poster if the total area of the poster is to be a minimum. (Verify, of course) (15 pts)

II Find the dimensions of the rectangle with greatest area which can be inscribed in a SEMI-circle of radius $5\sqrt{2}$. (15 pts)

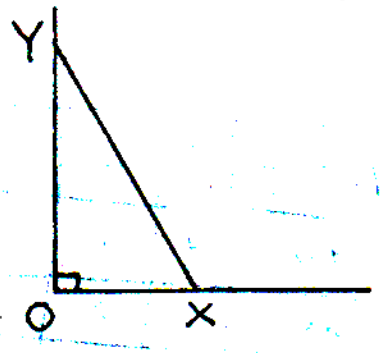
III The volume of a cone is increasing at the rate of 28π cubic inches per second. At the instant when the radius of the cone is 3 units, its volume is 12π units and the radius is increasing at one-half unit per second. (15 pts ea)

$$V = \frac{1}{3}\pi r^2 h$$

A) At the instant when the radius of the cone is 3 units, what is the rate of change of the area of its base?

B) At the instant when the radius of the cone is 3 units, what is the rate of change of its height?

IV A ladder 15 feet long is leaning against a building so that end X is on level ground and end Y is on the wall (See diagram). X is moved away from the building at the constant rate of one-half foot per second. (15 pts ea)



A) Find the rate in feet per second at which the length $\overline{OY}$ is changing when X is 9 feet from the building.

B) Find the rate of change in square feet per second of the area of triangle XOY when X is 9 feet from the building.

V The total cost $C(x) = 100 + \frac{1}{2}x^2 + \sqrt{x}$ of producing x units of a commodity. (10 pts tot)

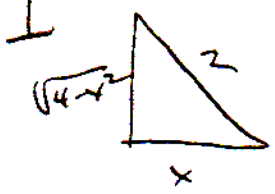
A) Find the average cost for producing 100 units.

B) Find the marginal cost for producing 100 units.

EXTRA CREDIT ---------- 5 pts

Continuing with Section III, at the instant when the radius of the cone is 3 units, what is the rate of change of the area of its base with respect to its height?

I



$A = \frac{1}{2} x\sqrt{4-x^2}$

$\frac{1}{2}\left(\sqrt{4-x^2} + \frac{x(-2x)}{2\sqrt{4-x^2}}\right) \overset{\text{set}}{=} 0$

$\frac{4-x^2-x^2}{2\sqrt{4-x^2}} = 0$

$A' = \frac{(\sqrt{2}-x)(\sqrt{2}+x)}{\sqrt{4-x^2}} = 0$

A': + (0) + (0) −, at $-\sqrt{2}$, 0, $\sqrt{2}$

$x = \sqrt{2}$ max of curve

$y = \sqrt{2}$

15

II

$\left.\frac{dy}{dx}\right| = ?$

$-1, 2\pi$

$xy + x\sin y = -2\pi$

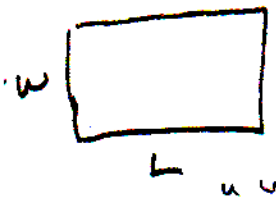
$y + x\frac{dy}{dx} + \sin y + x\cos y \frac{dy}{dx} = 0$

$2\pi - \frac{dy}{dx} + 0 - \cos 2\pi \frac{dy}{dx} = 0$

$2\pi - 2\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \pi$ 15

III



$l = 12$

$w = 5$

$\frac{dL}{dt} = 7$

$\frac{dw}{dt} = -3$

A) $A = wL$

$\frac{dA}{dt} = L\frac{dw}{dt} + w\frac{dL}{dt}$

$= 12(-3) + 5(7)$

$-36 + 35$

$\frac{dA}{dt} = -1$ 5

B) $P = 2w + 2L$

$\frac{dP}{dt} = 2\frac{dw}{dt} + 2\frac{dL}{dt}$

$= 2(-3) + 2(7)$

$-6 + 14$

$\frac{dP}{dt} = 8$ 5

C) $\frac{dd}{dt} = ?$

$d = w^2 + L^2$

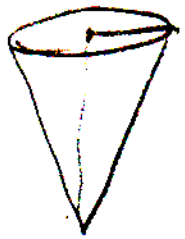
$2d\frac{dd}{dt} = 2w\frac{dw}{dt} + 2L\frac{dL}{dt}$

$\frac{dd}{dt} = \frac{w\frac{dw}{dt} + L\frac{dL}{dt}}{d}$

$\frac{dd}{dt} = \frac{5(-3) + 12(7)}{13}$

$\frac{-15 + 84}{13} = \frac{69}{13}$ 5

IV



$r = \frac{1}{3}h$

$\frac{dV}{dt} = 10$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{h^2}{9}\right)h$

$V = \frac{\pi h^3}{27}$

$1000 = \frac{\pi h^3}{27}$

$27000 = \pi h^3$

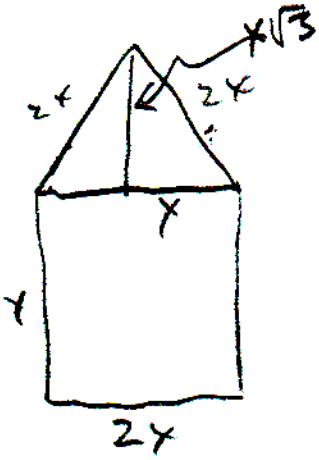
$h = \frac{30}{\pi^{1/3}}$

$\frac{dV}{dt} = \frac{\pi}{9}h^2\frac{dh}{dt}$

$10 = \frac{\pi}{9}\left(\frac{30^2}{\pi^{2/3}}\right)\frac{dh}{dt}$

$\frac{1}{10\pi^{1/3}} = \frac{dh}{dt}$ 20

V



$6x + 2y = 200$

$3x + y = 100$

$A = 2xy + x^2\sqrt{3}$

$A(x) = 2x(100 - 3x) + x^2\sqrt{3}$

$= 200x - 6x^2 + x^2\sqrt{3}$

$A' = 200 - 12x + 2x\sqrt{3} \overset{\text{set}}{=} 0$

$(2\sqrt{3} - 12)x = -200$

$x = \frac{100}{-\sqrt{3} + 6}$ 15

$A'' = -12 + 2\sqrt{3}$

$-12 + 2\cdot\sqrt{4} < 0 \therefore$ max.

VI

Let x = 1st #

$\frac{1}{2} - x$ = 2nd #

$f(x) = x^3 + \left(\frac{1}{2} - x\right)^2$

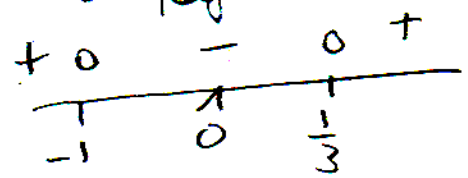
$f' = 3x^2 + 2\left(\frac{1}{2} - x\right)(-1)$

$= 3x^2 - 1 + 2x$

$3x^2 + 2x - 1$

$(3x - 1)(x + 1)$

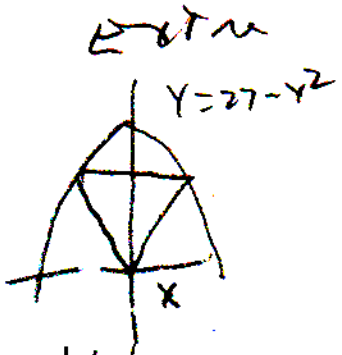
$x = \frac{1}{3}$ $x = -1$ Reject



A) 10 min at $-\frac{1}{3}, \frac{1}{6}$ f' test

B) 10 max at end pt id 1, 0 on the 2 #s

∴ Base of rectangle is $\frac{200}{-\sqrt{3} + 6}$



$A = \frac{1}{2}bh$

$A = \frac{1}{2}2x(27 - x^2)$

$= 27x - x^3$

$A' = 27 - 3x^2 \overset{\text{set}}{=} 0$

$x^2 = 9$

$x = 3$

$A'' = -6x < 0$ max ∴ f'' test

$A = \frac{1}{2}\cdot 6\cdot(27 - 9)$

$= 3(18)$

$= 54$

1989

	2	5	T
100	2	0	2
90	4	7	11
80	11	5	16
70	1	1	2
60	2	0	2
59	3	2	5
	80,87	83.07	81.74

103

40

Graded mainly on completion of problem. Majority of pts for the set up ie proper equation, Layout etc.

2 changes as shown this is probably adequate for today.

Honors ANALYSIS NAME Analysis

Convince ME that *YOU* know the concept ! PER _____ DATE Ch 4 App 5/16/89

Chapter 4 **APPLICATIONS** EXAM

I Among all right triangles with hypotenuse 2, determine (using calculus, of course) the side lengths of the one with the largest area. (Verify , as usual) (15 pts)
(HINT: *something* has to be x .)

II Find $\frac{dy}{dx}$ at $(-1, 2\pi)$ when $xy + x \sin y = -2\pi$ (15 pts)

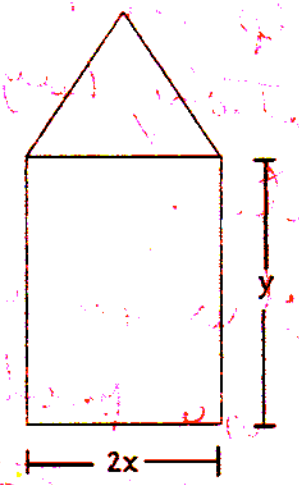
III The length of a rectangle is increasing at a rate of seven feet per second. The width is decreasing at a rate of three feet per second. When the length is twelve feet and the width is five feet, find the rate of change of : (25 pts)

A) the area B) The perimeter C) the length of a diagonal

IV A conical reservoir with radius equal to one-third its height is filling with water at a rate of ten cubic inches per second. How fast is the water level rising when there are one thousand cubic feet of water in the tank ?
The volume of a cone with height " h " and radius " r " is $\frac{1}{3}\pi r^2 h$. (20 pts)

V An equilateral triangle is placed on a rectangle as shown : The total *perimeter* (distance around exterior) is exactly 200 feet. For the figure to enclose a MAXIMUM area , show that the base of the rectangle would be : (15 pts)

$$\frac{200}{\sqrt{3} - 6} \leftarrow 6 - \sqrt{3}$$



Verify using the F " test .
(Hint : 30-60-90 triangle : 1 : 2 : $\sqrt{3}$)

VI Two numbers greater than or equal to zero have a sum of one - half. Find these two numbers if the cube of one of them plus the square of the other is to be :

A) a minimum B) a maximum (10 pts ea)

EXTRA CREDIT 5 points

Find the area of the largest isoceles triangle which can be drawn with one vertex at the origin and with the others on a line parallel to the x - axis and on the curve $y = 27 - x^2$.

I Let x = 1st number

$2x$ = 2nd #

$27-3x$ = 3rd #

$f(x) = x(2x)(27-3x)$

$= 54x^2 - 6x^3$

$f'(x) = 108x - 18x^2$

$18x(6-x) \overset{\text{set}}{=} 0$

$x \in \{0, 6\}$

$x=0$ reject $x>0$

$f''(x) = 108 - 36x \Big|_{x=6} = 108 - 216 < 0$

$\therefore$ max f'' test $x=6$ (25)

the numbers are 6, 12, 9.

II $\cos(xy) + 2x^2y^3 = 16\pi^3 + 1$

$-\sin xy\left[y + x\frac{dy}{dx}\right] + 4xy^3 + 6x^2y^2\frac{dy}{dx} = 0$ (5pts)

at $(-1, 2\pi)$

$-\sin(-2\pi)[\quad] + (-4)8\pi^3 + 6(4\pi^2)\frac{dy}{dx} = 0$

$[\quad] = 0$

$\frac{dy}{dx} = \frac{32\pi^3}{24\pi^2}$ (15)

$= \frac{4\pi}{3}$

III

$\frac{dr}{dt} = \frac{1}{4}$ $\frac{dA}{dt} = ?$

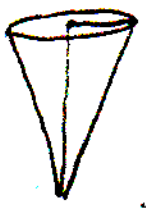
$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$= 2\pi \cdot 3\left(\frac{1}{4}\right)$

$\frac{dA}{dt} = \frac{3\pi}{2}$ (15)

IV



$h = 12R$ $\frac{dV}{dt} = 1$

$V = \frac{1}{3}\pi r^2 h$ $\frac{dh}{dt} = ?$

$r = \frac{h}{12}$

$V = \frac{1}{3}\pi\frac{h^3}{144}$ $h = 12R$

$h = R \cdot \frac{2}{3}$

$h = 8$

$\frac{dV}{dt} = \frac{\pi h^2}{144}\Big|\frac{dh}{dt}$

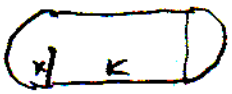
$r = \frac{2}{3}$

$h = 8$ (25)

$1 = \frac{64\pi}{144} = \frac{32\pi}{72} = \frac{4\pi}{9}\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{9}{4\pi}$

V



$2\pi x + 2K = 200$

$\pi x + K = 100$

$K = 100 - \pi x$

$A(x) = 2x(100 - \pi x)$

$= 200x - 2\pi x^2$

$A' = 200 - 4\pi x \overset{\text{set}}{=} 0$ $x = \frac{50}{\pi}$

$A'' = -4\pi < 0 \ \forall x$ $\therefore$ max f'' test

radius $\frac{50}{\pi}$ $K = 100 - \frac{50}{\pi}\pi = 50$

width of Rectangle $\frac{100}{\pi}$ (20)

Extra credit

$$x^2 + y^2 = 6$$

$$2x + 2y\frac{dy}{dx} = 0 \qquad \frac{dy}{dx} = -\frac{x}{y}$$

$$x + y\frac{dy}{dx} = 0$$

$$1 + \left(\frac{dy}{dx}\right)^2 + y\frac{d^2y}{dx} = 0$$

$$1 + \frac{x^2}{y^2} + y\frac{d^2y}{dx^2} = 0$$

$$\frac{y^2 + x^2}{y^2} + y\frac{d^2y}{dx^2} = 0$$

$$\frac{d^2y}{dx^2} = -\frac{y^2 + x^2}{y^3}$$

	2	5	
100	0	3	3
90	10	5	15
80	4	4	8
70	3	2	5
60	3	2	5
54	2	2	4
	81.3	81.8	81.5

100

35

The f's expected another exam — d'd guess they memorized one from 88 But didn't learn anything!

$\frac{1788}{22}$ $\frac{1472}{18}$

86

	2	5	
100	1	1	2
90	5	6	11
80	2	4	6
70	4	3	7
60	3	1	4
54	6	4	10
	70.9	77.9	74.2

102

5

Excellent exam — No changes.

ANALYSES Ch 4 App

HONORS ANALYSIS
Convince ME that YOU know the concept !
Copy original problem. SHOW ALL WORK !

NAME _____

PER _____ DATE 1990

CHAPTER 4 APPLICATIONS EXAM

I Find three positive numbers such that one is twice another and the sum of all three is 27 AND their product is a maximum. (25 pts)

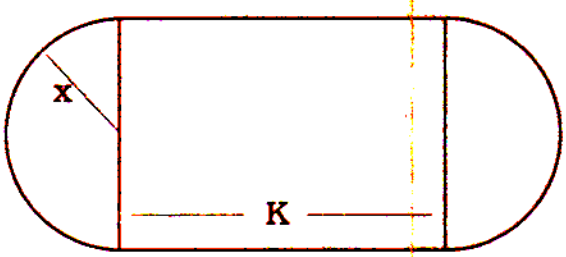
II Find $\frac{dy}{dx}$ at $(-1, 2\pi)$ when $\cos(xy) + 2x^2y^3 = 16\pi^3 + 1$ (15 pts)

III A stone is dropped into a still pond. Concentric ripples are spreading out. The radius of the disturbed region is increasing at a rate of one-fourth of a foot per second. At what rate does the area of the disturbed region increase when its radius is three feet? (15 pts)

IV A conicle icicle whose height is always 12 times the radius of its base is being formed by some dripping water. If the volume is increasing at the rate of 1 cm/hr, at what rate is the height increasing when the radius is two-thirds of a centimeter? (25 pts)

(The volume of a cone with height "h" and radius "r" is $\frac{1}{3}\pi r^2 h$.)

V An indoor physical fitness room consists of a rectangular region with a semicircle on each end. If the perimeter of the room is to be a 200 meter running track, find the dimensions that will make the area of the ***rectangular*** region as large as possible. See the diagram. (Hint: Write K in terms of x then describe the area of the rectangle – verify, of course.) (20 pts)



EXTRA CREDIT 5 pts

The first derivative of y with respect to x is sometimes written: $\frac{dy}{dx}$ and is roughly the same as $F'(x)$.

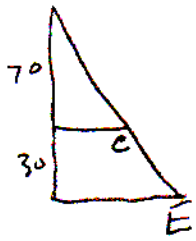
The second derivative of y WRT x is then written: $\frac{d^2y}{dx^2}$ and is roughly the same as $F''(x)$.

The first derivative of y WRT x *TIMES* the first derivative of y WRT x is written: $\left(\frac{dy}{dx}\right)^2$

Without using the $\frac{u}{v}$ quotient rule or any negative exponents, find $\frac{d^2y}{dx^2}$ if $x^2 + y^2 = 6$.

Your final answer must NOT contain a $\frac{dy}{dx}$.

I



$\frac{c}{70} = \frac{E}{100}$

$\frac{10}{7} c = E$

a) $\frac{dc}{dt} = 2$

$\frac{dE}{dt} = ?$

$\frac{10}{7}\frac{dc}{dt} = \frac{dE}{dt}$

$\frac{10}{7}(2) = \frac{dE}{dt}$

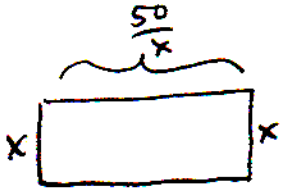
$\frac{20}{7} = \frac{dE}{dt}$

The shadow moves at a constant rate regardless how far she is since there is no c in the equation

Thus a. $\frac{20}{7}$ ft/sec

b. $\frac{20}{7}$ ft/sec

II



$\frac{dx}{dt} = 2$

$\frac{dp}{dt} = ?$

a) $p = 2x + \frac{100}{x}$

$\frac{dp}{dt} = 2\frac{dx}{dt} - \frac{100}{x^2}\frac{dx}{dt}$

$= 2(2) - \frac{100}{(5)^2}(2)$

$= 4 - \frac{100}{25}(2)$ The perimeter is shrinking at 4"/sec

$= 4 - 8 = -4$

b) $2(2) - \left(\frac{100}{100}\right)(2)$

$4 - 2 = 2$

The perimeter is growing at 2"/sec

c) $\frac{dp}{dt} = \left(2 - \frac{100}{x^2}\right)\frac{dx}{dt} = 0$

$\left(2 - \frac{100}{x^2}\right)(2) = 0$

$2 = \frac{100}{x^2}$

$x^2 = 50$

$x = 5\sqrt{2}$

$\frac{10}{\cancel{50}} \quad \sqrt{2}$

$\frac{}{\cancel{x}\sqrt{2}} \quad \sqrt{2}$

$5\sqrt{2}$

dimensions when shrinking stopped

$5\sqrt{2} \times 5\sqrt{2}$

III $P = x^2 - 195x + 24000$

$C = 12000x + 200000$

a) Revenue $= x^3 - 195x^2 + 24000x$

b) Profit $= x^3 - 195x^2 + 24000x - (12000x + 200000)$

$= x^3 - 195x^2 + 12000x - 200000$

c) $P' = 3x^2 - 390x + 12000$

$3(x^2 - 130x + 4000)$

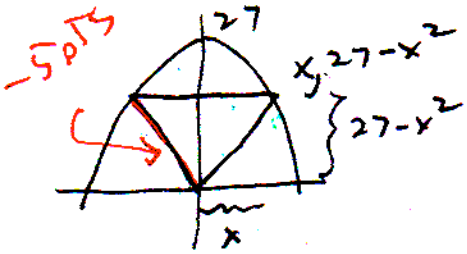
$3(x-50)(x-80) \overset{Set}{=} 0$

$x \in \{50, 80\}$

P' : $+ \; 0 \; - \; 0 \; +$ on $0, 50, 80$

The company should sell 50 units max, f' test.

IV



$A = \frac{1}{2}bh$

$A = \frac{1}{2}(2x)(27-x^2)$

$A(x) = 27x - x^3$

$A'(x) = 27 - 3x^2 \overset{Set}{=} 0$

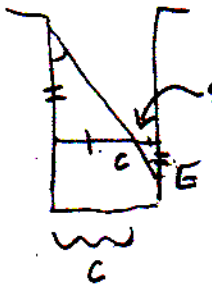
$3(3-x)(3+x) = 0$

$+ \; 0 \; -$ on $0, 3$

Max area at $x = 3$ f' test

$A(3) = 81 - 27 = 54$ sq units.

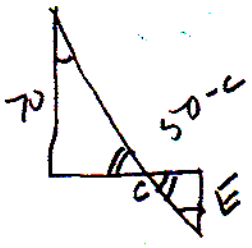
Extra.



$\frac{dc}{dt} = 2 \quad \frac{dE}{dt} = ?$

$\frac{E}{70} = \frac{50-c}{c}$

$Ec = 3500 - 70c$



$c\frac{dE}{dt} + E\frac{dc}{dt} = -70\frac{dc}{dt}$

$\frac{E}{70} = \frac{40}{40}$

$E = \frac{70}{4} = \frac{35}{2}$

$40\frac{dE}{dt} + \frac{35}{2}(2) = -70(2)$

$40\frac{dE}{dt} = -175$

$\frac{dE}{dt} = -\frac{175}{40} = -\frac{35}{8}$

The shadow is going up the wall at $\frac{35}{8}$ ft/sec.

91

	2	5	T
100	1	0	1
90	6	1	7
80	1	1	2
70	2	6	8
60	3	5	8
59	9	8	17

101

15

$\frac{1548}{22}$ $\frac{1250}{20}$ $\frac{2798}{42}$

70.36 62.5 66.62

Excellent Exam —
Floundering Students

Next Year Try putting all
the on Boards & making students
explain to entire class. —
Yes! that's a good idea.

Only change as indicated
on review sheet.

Since I do want
better understanding
I will give
back ½ of
Lost pts for
the top Narrative
it increases
of a page long
and very
detailed and
accurate

96

	5	6	T
100↑	5	2	7
90	4	3	7
80	3	2	5
70	2	1	3
60	4	1	5
59	6	11	17

110

0

5↑

1754/22 = 79.73

1219/18 = 67.72

2973/40 = 74.33

4 of the 17 got zeros. They simply didn't do anything. They failed all along and finally realized the were getting what they earned. They were not included in the STATS.

I gave 5 pts "style" if they pointed out that no computations was required for part b on I because the length AC dropped out of the expressions for $\frac{dS}{dt}$.

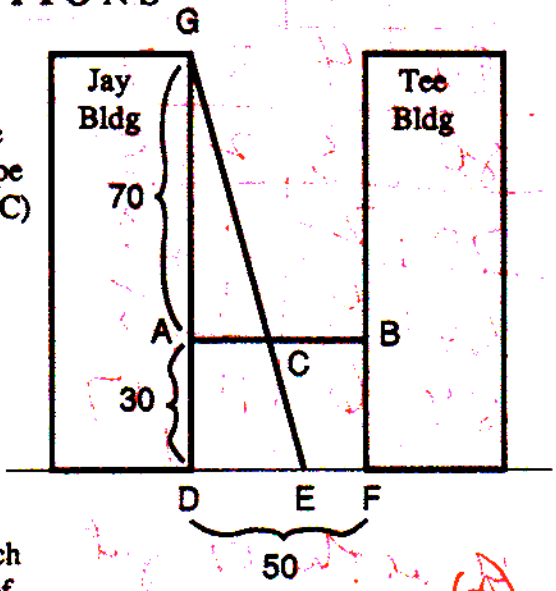
Honors Analysis Name ANL Ch 4 HW

Copy the original problem.
Convince *me* that **you** understand the concept. Per _____ Date 9/19/96

CHAPTER #4 APPLICATIONS

Each section is worth 25 points.

I A tightrope is stretched 30 feet above the ground between the Jay and the Tee buildings, which are 50 feet apart. A tightrope walker, walking at a constant rate of 2 feet per second (point C) from point A to point B, is illuminated by a spotlight 70 feet above point A, as shown in the diagram. How fast is the shadow (point E) of the tightrope walker's feet moving when she is:



A) 10 feet from point A?
B) midway between the buildings?

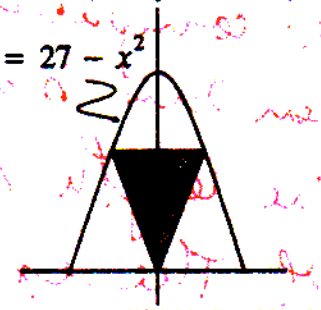
II Two parallel sides of a rectangle are being lengthened at the rate of 2 in/sec. while the other two sides are shortened in such a way that the figure remains a rectangle with constant area of 50 sq. in. What is the rate of change of the perimeter when the length of an increasing side is:

A) 5 inches?
B) 10 inches?
C) What are the dimensions when the perimeter ceases to decrease?

III A certain business had some studies done and found the relationship of $p(x)$, the price **per unit**, and x, the number of units which would be sold was $p(x) = x^2 - 195x + 24000$. The **total** production cost was $c(x) = 12000x + 200000$. Due to size limitations, the business could produce 60 items a month. The real question was, could the company make more profit if it produced less than 60 items per month? (hint: 8 + 5 = 13 and 8•5 = 40)

A) What is the revenue function (R=xp)?
B) What is the profit function(Profit = R – Costs)?
C) What should be the production level so the company will have maximum profits? (verify, as usual)

IV Find the area of the largest isosceles triangle that can be drawn with one vertex at the origin and with the others above the vertex on a line parallel to the x-axis and on the parabola $y = 27 - x^2$.



Extra Credit ——— ***5 pts*** ———

Continuing the tightrope walker's story, (problem I); how fast is the shadow moving *up* the side of building Tee when she (point C) is 10 feet from point B?

I $V = \frac{4}{3}\pi r^3$ a) $\frac{dV}{dt} = ?$

$\frac{dr}{dt} = \frac{4}{100}$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}\Big|_{r=10}$

$= 4\pi(100)\cdot\frac{4}{100}$

$= 16\pi$

The volume increases at 16π cm^3/sec.

b) $36\pi = \frac{4}{3}\pi r^3$

$\frac{3}{4}\cdot 36 = r^3$

$27 = r^3$

$r = 3$.

$A = \pi r^2$ $\frac{dA}{dt} = ?$

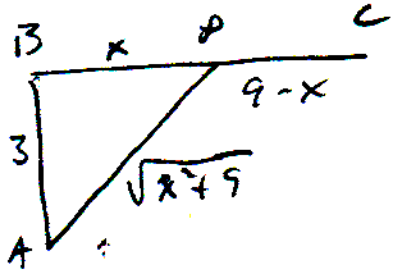
$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$= 2\pi(3)\frac{4}{100}$

$= \frac{6\pi}{25}$

The area of the circle is increasing at $\frac{6\pi}{25}$ centimeters per second.

II



$RT = D$

$T = \frac{D}{R}$

Time x-county $= \frac{\sqrt{x^2+9}}{80}$

Time Road $= \frac{9-x}{100}$

$f(x) = \frac{\sqrt{x^2+9}}{80} + \frac{9-x}{100}$

$f'(x) = \frac{1}{80}\cdot\frac{x}{\sqrt{x^2+9}} - \frac{1}{100} \overset{set}{=} 0$

$10x - 8\sqrt{x^2+9} = 0$

$100x^2 = 64(x^2+9)$

$64x^2 + 576$

$36x^2 = 576$

$x^2 = 16$

$x = 4$

Point P is located 4 miles to the Right of Point B

$f' = \frac{x}{80\sqrt{x^2+9}} - \frac{1}{100}$

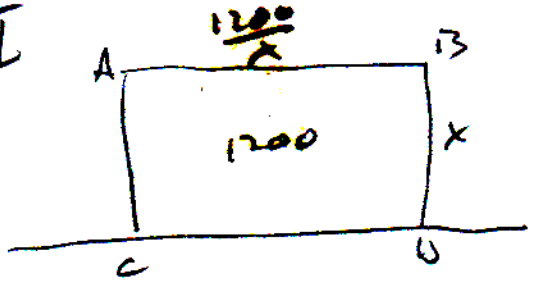
$f'' = \frac{80\sqrt{x^2+9} - x\cdot\frac{80\cdot 2x}{2\sqrt{x^2+9}}}{(80)^2(x^2+9)}$

$f'' = \frac{80(x^2+9) - 80x^2}{80(x^2+9)^{3/2}}$

$f'' = (x^2+9)^{\frac{1}{2}}$

$f''(4) > 0 \therefore$ Min f'' test.

III



a) $y = 3.2x + 3\cdot\frac{1200}{x} + \frac{3}{2}\cdot\frac{1200}{x}$ Shared fence

$y = 6x + \frac{3600}{x} + \frac{1800}{x}$

$y = 6x + \frac{5400}{x}$

b) $y' = 6 - \frac{5400}{x^2} \overset{\text{set}}{=} 0$

$6x^2 = 5400$

$x^2 = 900$

$x = 30$

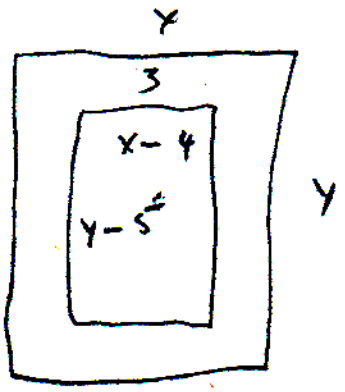
$y'' = +\frac{10800}{x^3}\Big| > 0$ ∴ min f'l cost.

$x=30$

$y = 6(30) + \frac{1800}{30} = \$360$

minimum cost is \$360.

IV



$(x-4)(y-5) = 320$

$y = \frac{320}{x-4} + 5$

a) $A = lw$

$= xy$

$A(x) = x\left[\frac{320}{x-4} + 5\right]$

$= \frac{320x}{x-4} + 5x$

$A'(x) = \frac{(x-4)320 - (320x)}{(x-4)^2} + 5$

$= \frac{320x - 1280 - 320x}{(x-4)^2} + 5$

$A'(x) = \frac{-1280}{(x-4)^2} + 5$

$\frac{-1280}{(x-4)^2} + 5 \overset{\text{set}}{=} 0$

$5(x-4)^2 = 1280$

$(x-4)^2 = 256$

$x-4 = 16$

$x = 20$

$A'' = \frac{2560}{(x-4)^3}\Big| > 0$ ∴ min f'l cost.

$x=20$

$y = \frac{320}{16} + 5$

$20 + 5$

$y = 25$

Dimensions are 20×25.

Extra credit

$\frac{dv}{dt} \overset{\text{set}}{=} \frac{ds}{dt}$

$4\pi r^2 \frac{dr}{dt} = 2\pi r \frac{dr}{dt}$

$r = \frac{1}{4\pi}$

$r = \pm\frac{1}{2\sqrt{\pi}}$

$r = \frac{1}{2\sqrt{\pi}}$ when $r = \frac{1}{2\sqrt{\pi}}$ the rate of change of the volume of the sphere.

93

	2	5	T
100	1	0	1
90	0	3	3
80	5	4	9
70	3	1	4
60	2	4	6
59	10	7	17

85

	5	6	T	
100	0	1	1	101
90	3	3	6	
80	4	3	4	
70	5	0	5	
60	3	2	5	
59	4	1	5	

1261/21 = 60.0

1151/18 = 63.9

2412/39 = 61.8

1152/16 = 72

824/10 = 82.4

45 76.

One minor typo to change. Poor Grades because most were NOT Preparing material before & told them about problems. 5 of 10 to whom gives up wrong ago.

better results but still too many low grades. This year I assigned 3-5 probs and required "explain wrong". They didn't seem to make much difference.

5/15/98

100	4
90	2
80	5
70	67
60	0
59	1

104

$\frac{1566}{19}$ 53

82.4

The answers — "Just policy is my given answers" is not they don't prove min / max, they don't show necessity of item in subsets.

so this method is ok.

Honors Trigonometry Name _____ AN L Ch 4 Ap

Copy original problem. Per _____ Date 5/15/98

Convince *me* that you understand the concept!

Chapter 4 – Applications Exam

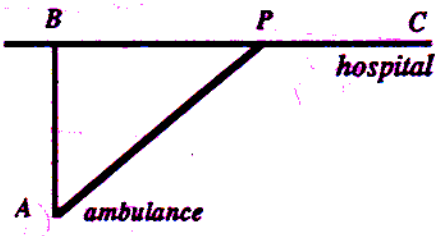
I The radius r of a sphere is increasing at a constant rate of 0.04 cm per second.

note: $V_{sphere} = \frac{4}{3}\pi r^3$ (25 pts tot)

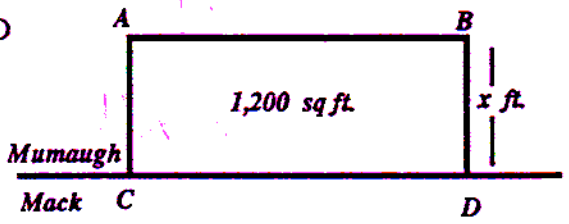
a) At the time when the radius of the sphere is 10 cm, what is the rate of increase of its volume?

b) The cross section through the center of the sphere is a circle. At the time when the volume of the sphere is 36π cubic centimeters, what is the rate of increase of the area of the circle?

II The diagram shows a straight road BC running due East. A four wheel drive ambulance is in open country at A, 3 miles due south of B. It must reach a hospital at C, 9 miles due East of B, as quickly as possible. The ambulance can travel at 80 mph in the open coun try and 100 mph along the road. The driver intends to proceed in a straight line to some point P on the road and then to continue along the road to the hospital at C. He needs to choose P so that the total time for the journey from A to P to C is a minimum. Remember, $R \cdot T = D$. Show that P is 4 miles East of B. (Verify using f'' test.) (25 pts tot)



III Rancher Mumaugh wishes to fence off a rectangular yard ABCD of area 1200 sq. ft. from a grazing field, as in the diagram, with the side CD against the property of his neighbor, L. Mack. Fencing costs \$3 per foot and L. Mack has agreed to pay half the cost of the side CD. Let \$y be the cost to Mumaugh of fencing the yard and let x feet be the length of BD.

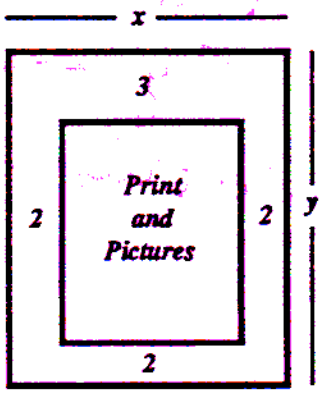


(25 pts tot)

a) Obtain a formula for y in terms of x.

b) Find the minimum cost to Mumaugh of fencing the yard.

IV "Calculus for Toddlers", a children's picture book by Dr. Suess with forward written by Charleton Heston, is being designed so that each page contains 320 cm² of print and pictures surrounded conpletely by a white border as shown. Each page is to have a border of width 2 cm at the bottom an on each side, as well as a border of width 3 cm at the top.
Let the width of a page be x cm and its length by y cm. (25 pts tot)



a) Show that the area A of one such page is given by: $A(x) = x\left[5 + \frac{320}{x-4}\right]$

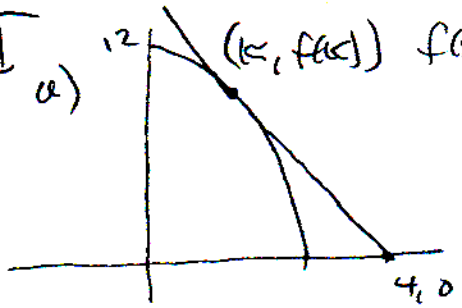
b) Show that the page which meets all the printing requirements given and which has the smallest area is 20 cm by 25 cm.

Extra Credit ---------- 5 pts ----------

Referring to problem in section I: What is the radius at the time when the volume and the radius of the sphere are increasing at the same numerical rate?

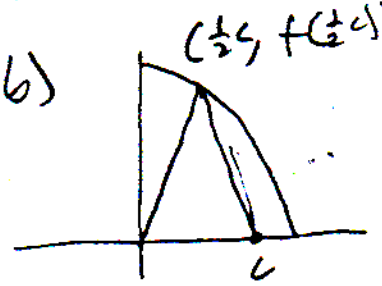
I

a)



$f(k) = 12 - k^2$

b)



$A = \frac{1}{2}bh$

$= \frac{1}{2}c\left(12 - \left(\frac{c}{2}\right)^2\right)$

$= \frac{1}{2}c\left(12 - \frac{c^2}{4}\right)$

$= 6c - \frac{c^3}{8}$

$f(k) = 12 - k^2 \quad f'(k) = -2x\Big|_{x=k} = -2k$

$A' = 6 - \frac{3c^2}{8} \overset{\text{set}}{=} 0$

$\frac{12 - k^2 - 0}{k - 4} \overset{\text{set}}{=} -2k$

$3c^2 = 48$

$c^2 = 16$

$c = 4$

$12 - k^2 = -2k^2 + 8k$

$A'' = -\frac{3c}{4} < 0 \therefore$ max. f' test

$k^2 - 8k + 12 = 0$

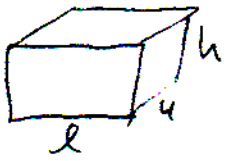
$(k - 6)(k - 2) = 0$

$k = \{6, 2\}$

Reject $k > 6$ since $k < 2\sqrt{3}$

$k = 2 \therefore$

II



$4hl = 36$

$4l = 9$

$l = \frac{9}{4}$

base $4l \cdot 10 = 40l$

2 sides $2 \cdot 4h \cdot 5 = 40h$

2 front $2hl \cdot 5 = 10hl$

$C = 40l + 40h + 10hl$

$C(h) = \frac{360}{4h} + 40h + 90$

$C'(h) = -\frac{360}{4h^2} + 40 \overset{\text{set}}{=} 0$

$h^2 = 9$

$h = 3$

$C''(h) = \frac{720}{h^3} > 0 \therefore$ min f'' test.

$C(3) = 120 + 120 + 90$

The cost for the box is $330.

III

~~a) $\overline{OA} = \overline{OB}$ become both are radius of the same circle.~~

b) $\overline{OA} = 4\sqrt{2}$ because the side of the square is also the hypotenuse of RT triangle $\therefore$ = length.

c) $A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$= 2\pi \cdot 3 \cdot 6 = 36\pi$

d) by Square has side $2r \therefore$

$A = 4r^2$

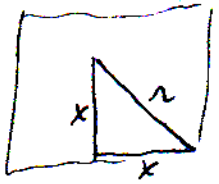
$\frac{dA}{dt} = 8r \frac{dr}{dt}$

$= 8 \cdot 4 \cdot 6$

$32 \cdot 6 = 192$

III

e) Sin Square



$2x^2 = r^2$

$x = \frac{r}{\sqrt{2}}$ side of small square

$2\frac{r}{\sqrt{2}} = r\sqrt{2}$

$A = 2\pi r^2$

$\frac{dA}{dt} = 4\pi r \frac{dr}{dt}$

$= 4 \cdot 5 \cdot 6$

$= 120$

f) $P_B = 8r$

$\frac{dP}{dt} = 8\frac{dr}{dt}$

$= 48$

$P_S = 4\sqrt{2}r$

$\frac{dP}{dt} = 4\sqrt{2}\frac{dr}{dt}$

$= 24\sqrt{2}$

$\frac{48}{24\sqrt{2}} = \frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

Per 6 sec, 8 changes $\sqrt{2}$ times faster than Sm's rg collects r.

g) $\frac{\text{Lg-circle}}{\text{circle-Sm}}$

$\frac{4r^2 - \pi r^2}{\pi r^2 - 2r^2}$

$\frac{4 - \pi}{\pi - 2}$

$\approx .752$

IV

$\frac{d\theta}{dt} = 4\pi$

$\sin\theta = a$

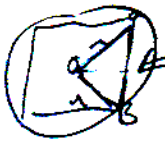
$\cos\theta \frac{d\theta}{dt} = \frac{da}{dt}$

$\cos\frac{\pi}{3} \cdot 4\pi =$

$\frac{1}{2} \cdot 4\pi =$

2π rad/min.

III another version



$\sqrt{2}r^2 = r\sqrt{2}$ = side of small square

$A = (r\sqrt{2})^2 = 2r^2$

$\frac{dA}{dt} = 4r\frac{dr}{dt}$

$= 4 \cdot 5 \cdot 6$

$= 120$

EXTRA.

Draw a graph with the horizontal axis the hours starting with 7 am and progressing to 2 pm. Let the vertical axis be distance to the top of the mountain.

Plot a typical graph up the mountain. Next plot a typical graph down the mountain. Where the two graphs cross will be the exact same spot on the trail at the exact same time of day!

Second version assume one group is at the top of the mountain and another at the bottom. During the day, the two groups will meet on the path. This is that point we are interested in.

1999

Question #1 needs better wording

The problem with the students is that they waited to do the problems until the day they were due and copied from the boards. They only really focused on the one prblm they did so couldn't practice the organization necessary to pass this ~~Exam~~ exam

	4	3	T
100	0	0	0
90	0	1	1
80	1	3	4
70	3	5	8
60	4	4	8
59	6	6	12

93

10

$\frac{867}{15}$ $\frac{1209}{18}$ $\frac{2076}{33}$

57.8 67.2 62.9

Honors Analysis Name AJL Ch 4 Apps

Copy original problem. Per _____ Date 99

Convince *me* that **you** understand the concept!

Chapter 4 Applications Exam

I

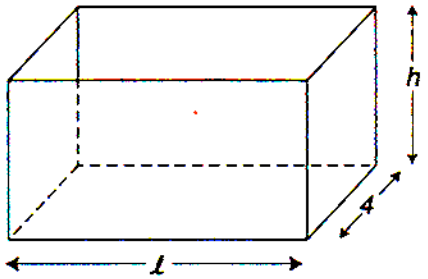
Let $f(x)=12-x^2$ for $x\geq 0$ and $f(x)\geq 0$. (15 pts ea)

a) The line tangent to the graph of $f(x)$ at the point $(k,f(k))$ intercepts the x-axis at $x=4$. Draw a useful sketch and label the points stated and draw the tangent line. Determine the value of k.

b) An isosceles triangle whose base is the interval from $(0,0)$ to $(c,0)$ has its vertex on the graph of $f(x)$. Draw a new picture which includes the triangle. Be sure all significant points are clearly labled. For what values of c does the triangle have maximum area? Justify, of course.

II

A tank with a rectangular base and rectangular sides is to be open at the top. It is to be constructed so that its width is 4 meters and its volume is 36 cubic meters. If building the tank costs \$10 per square meter for the base and \$5 per square meter for the sides, what is the cost of the least expensive tank?
(30 pts)



III

composed of two squares and circle.

Consider the figure on the right. Given $\frac{dr}{dt}=5$ where r is the radius of the circle. (30 pts tot)

the circle has center marked O A and C are mid pts of the sides of the square A expands at 6 units per second and the square grows to remain in contact.

a) Explain how you know that $\overline{OC}=\overline{OB}$. of the squares.

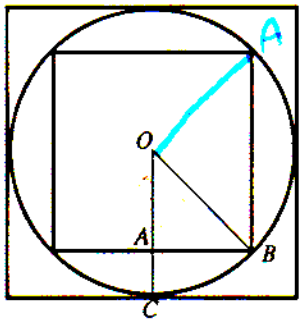
b) Explain how you know that $\overline{OA}=\overline{AB}$.

c) How fast is the area of the circle changing when $r = 3$?

d) How fast is the area of the large square changing when $r = 4$?

e) How fast is the area of the small square changing when $r = 5$?

f) How many times faster is the rate of change of the perimeter of the outside square as compared to the perimeter of the inside square when $r = 6$?

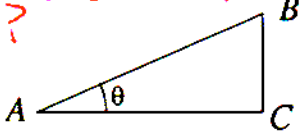


g) Let area p be the area outside of the circle and inside the large square. Let area q be the area outside of the inside square and inside the circle. Compare area p to area q. Round your answer to 3 decimal places.

what is the ratio of area p to area q?

IV

Let triangle ABC be a right triangle with a fixed hypotenuse = 1 meter. If θ is increasing at the rate of 4π radians per minute, find the rate at which the side "a" is increasing when $\theta=\pi/3$. (10 pts)



Extra Credit ---------- 5 pts ----------

A group of hikers leave the foot of a mountain at 7:00 A.M. and hikes to the top arriving by 2:00 P.M. They spend the night on the mountain and hike back down the next morning, again leaving at 7:00 A.M. but arriving back at base camp at exactly at noon. Assuming that only one trail exists, did the hikers pass any point at the same *time* on both days? Explain.

I Given:

	f	f'	g	g'	h	h'
0	1	2	2	2	1	2
1	2	1	1	0	2	1
2	2	1	2	1	0	0

evaluate the following:

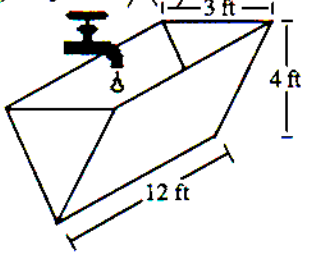
Of course, we know ... $(f \circ g)(x) = f(g(x))$

a) $(f \circ g)'(0)$ b) $(f \circ g)'(2)$ c) $(f \circ h)'(0)$ d) $(g \circ f)'(1)$

e) $(f \circ g)'(1)$ f) $(g \circ g)'(1)$ g) $(g \circ f)'(2)$ h) $(g \circ f \circ h)'(2)$

i) $(f \circ h \circ g)'(1)$ j) $(g \circ h \circ f)'(0)$ k) $(f \circ f \circ f)'(0)$

II The length of the water trough is 12 feet. It was made by bending some metal in the shape of a V and then the ends were soldered to the trough to make it water tight. The cross section of the trough (i.e. each end of the trough) is an isosceles triangle with measurements as shown in the picture. The water faucet delivers water at 3 cubic feet per minute.



A) Assume that the trough has already been filled to a depth of 2 feet (as measured from the apex of the triangle to the surface of the water.) Your watch reads 2:20 in the afternoon. What time will it be when the trough is completely full?

B) How fast was the depth changing at 2:38?

C) How fast was the depth changing at 2:40?

III A package to be sent in the mail can have a combined girth (distance around) plus length of at most 72 inches. What is the volume of the largest box (maximum volume), with a square base, which can be sent?

IV Find the extrema of $f(x) = 4x^3 - 12x + 7$ for $[-2, 0]$

V Along the graph of the equation $y = x^3 - 6x^2 + 3x + 5$ both the ordinate "y" and the slope (of the tangent) "m" change, but generally at different rates. Find the point (or points), if any, where the ordinate and the slope are momentarily changing at the *SAME* rate.

VI Water flows at the rate of 2 cubic feet per minute into a vessel in the form of an inverted right circular cone of altitude 2 feet and base radius 1 foot. At what rate is the surface of the water rising when the vessel is one-eighth full?

VII Since 1980 the enrollment at Vacaville High School is approximated by the formula given as "V". The student population of Wood High School is given by the formula "W". (a) What will be the maximum (or minimum) enrollments for each school? (b) When will it occur? (t is the number of years since 1980 so 1981 is $t = 1$) (c) What are the current enrollments? (d) When, if ever, did (or will) the student populations be the same? (e) Give a reason why the growth rates are reasonable. (f) During which year will both schools have the same number of students? (Please be accurate to the nearest year.)

$$V(t) = 1100 + (t - 17)^2 \qquad W(t) = 1400 - 1000(t + 7)^{-1}$$

VIII One leg (AC) of right triangle ACB increases at the rate of 2 in/sec while the other leg decreases at the rate of 3 in/sec. Determine how fast the hypotenuse is changing AND state whether it is getting longer or shorter when leg BC = 6 and leg AC = 8. What about the area of the triangle at the same time?

IX Find the altitude of the isosceles triangle with largest area which can be inscribed inside a circle with radius 5.

X A man is flying a kite which is a constant 50 feet above the ground. A horizontal wind is blowing the kite away from the man. When 130 feet of string has been let out, the kite is pulling the string out at a rate of 20 ft/sec. What is the wind velocity?

XI We are considering investing a large sum of money. Our broker brought us the figures for two companies. Acme Unlimited and Baker Industries. The net assets are in units of 1 billion dollars.

$$\text{Acme } A(t) = 3t(t+1)^{-1} \qquad \text{Baker } B(t) = t^2(t+1)^{-1} + 1$$

A) Find the assets for each company one year from today (ie. when $t = 1$).

B) Which company should we invest with? Why?

C) Determine the growth rate (ie. the rate of change in the assets = growth rate) for each company.

D) Evaluate each of these at $t = 1$.

E) Are you now able to decide which company is the best choice (from the investor's point of view)? Why?

F) Let's consider how the "growth rate" is changing. Since the first derivative of assets is growth rate, it seems logical that the first derivative of growth rate will tell us how active the growth rate is. So find the second derivative of each function.

G) Evaluate both at $t = 1$.

H) When the second derivative is positive, we say the business is growing (ie. the growth rate is accelerating). (We are holding the gas pedal of the car completely down.) When the second derivative is negative we say the growth rate is decelerating. (To continue the analogy, this is like removing your foot from the gas pedal. The car is still traveling but is slowing at the same time.) We have seen that the assets of both companies will be the same 1 year from now, and their growth rates are the same. However, we can see that one of the businesses will increase as time marches on while the other ...

I) Should we choose a company with a currently high growth rate that is decreasing slowly, or one with a smaller growth rate that is increasing slowly? Why?

J) Speculate on the idea that the business which is decreasing has currently a higher return than the business which is increasing -- that is -- which would be the best choice? Why?

XII The (x, y) coordinate of a point P is given by: $x = t + \frac{1}{t}$; $y = t - \frac{1}{t}$

A) Find $\frac{dy}{dx}$ (Hint: you are using the definition of *Chain Rule*)

B) Find $\frac{d^2y}{dx^2}$ (This is the second derivative of y with respect to x. Answer is **NOT** -1)

C) Combine the two given equations in such a fashion that the variable t is eliminated.

D) What conic section is this? Draw its graph -- (Ask someone in Alg II -- They can help.)

XIII The sum of two nonnegative numbers is 1.

A) The sum of the square of the first and the cube of the second is a maximum. Find both numbers.
B) The sum of the square of the first and the cube of the second is a minimum. Find both numbers.

XIV Given rectangle $ABCD$ has 10 sq ft of area. Point E is the midpoint of side BC. Determine the dimensions of $ABCD$ such that the distance from A to E is a minimum.

XV Given the parabola: $y = 16 - x^2$. A rectangle has two of its vertices on the x-axis and the other two above the x-axis and on the parabola.
A) Of all such possible rectangles, find the dimensions of the one with maximum area.
B) Of all such possible rectangles, find the dimensions of the one with maximum perimeter.

XVI To study the effect of production levels on costs, economists use the "average cost function". The average cost function (called "C-bar") is written $\overline{C} = \frac{C(x)}{x}$ where $C(x)$ is the total cost function.

A company estimates that the cost (in dollars) of producing x units of a certain product is given by:

A) $C(x) = 800 + .4x + .0002x^2$

1. Find the production level that minimizes the average cost per unit.
2. Compare the answer to part 1 with the average cost when
   a) 4000 units are produced.
   b) 1000 units are produced.

B) $C(x) = x^3 - 6x^2 + 13x$

1) Find the value of x where the average cost is a minimum.
2) For the value found in part 1, show that the marginal cost and average cost are equal.

C) $C(x) = 2x^2 + 5x + 18$

1) Find the value of x where the average cost is a minimum.
2) For the value found in part 1, show that the marginal cost and average cost are equal.

D) Show that the average costs is a minimum at the value of x where the average costs equals the marginal cost.

XVII A business, in marketing a certain item, has discovered through trial and error experiments in several test areas that the demand this item has is given by the equation: $x = \frac{2500}{q^2}$ where x is the number of units produced and q is the price per unit. Assuming that the total revenue function is normal (i.e. $R = x\,q$) and the cost function is $C(x) = .5x + 500$, find the price per unit that yields a maximum profit.

XVIII The manufacturer of Sweet Bippies charges \$90 per sweet bippy while the cost per sweet bippy is \$60. To encourage larger orders from the distributors, the manufacturer will reduce the price per Sweet Bippy by 10 cents for each sweet bippy ordered in excess of 100. (For example, there would be a charge of \$88 per bippy for an order size of 120). What would be the ideal order size, from the manufacturer's point of view. What would happen if an order came in which was twice as big as that?

XIX The vertices of a triangle are $(0, 0)$, $(x, \cos x)$, $(\sin^3 x, 0)$, where $0 < x < \frac{\pi}{2}$.

a) If $A(x)$ represents the area of the triangle, write the formula for $A(x)$.

b) Find the value of x for which $A(x)$ is a *maximum*. Justify, of course.

c) What is the permeter of triangle when the area is at its maximum?

XX Widgets-R-Us has a revenue function $R(x) = 52500x + 450x^2 - x^3$. What production level will yield a maximum revenue?

XXI A lighthouse is on an island one mile from a straight shoreline. Suppose that the light is turning at the rate of two revolutions per minute in the counterclockwise direction.

a) How fast is the light beam moving along the shore at the instant it makes an angle of 45° with the shoreline.

b) How fast is the light beam moving at the instant the angle is 90°?

XXII Each of the following functions has interesting but different behavior at $x = 2$. Graph each function (You may copy the picture from your grapher if you provide the window settings.) Tell at , $x = 2$, whether the function (list all that apply)

has a function value
has a limit
is continuous
is concave down

has a tangent line
is differentiable
has an asymptote
is increasing

has a cusp
has extreme value
is concave up
is decreasxing

1) $f(x) = (x-2)^2 + 1$

2) $f(x) = (x-2)^3 + 1$

3) $f(x) = (x-2)^4 + 1$

4) $f(x) = (x-2)^{\frac{1}{3}} + 1$

5) $f(x) = (x-2)^{\frac{2}{3}} + 1$

6) $f(x) = (x-2)^{\frac{1}{2}} + 1$

XXIII Amos Take analyzes $f(x) = x^2$, defined for all x in the closed interval $[-2, 3]$. He finds that $f''(-2) = 2$ and concludes that the graph is concave upward there, and thus that there is a local minimum at $x = -2$. Explain to Amos the error in his thinking.

XXIV A rectangle is inscribed in the region bounded by the x-axis and the parabola $y = 36 - x^2$. The base of rectangle is on the x-axis.

a) Find the length and width of the rectangle of greatest area.

b) Find the length and width of the rectangle of greatest perimeter.

c) Why does the rectangle of greatest area have the greatest perimeter?

Jake's Hamburger Stand

Useful Definitions:

DEMAND FUNCTION: A mathematical relationship between the number of units demanded, x, and the price per unit, Q.

REVENUE FUNCTION: A description of the gross income determined by the number of items sold multiplied by the price per unit.

COST FUNCTION: A description of the costs related to the sales of x units.

PROFIT FUNCTION: The difference between the gross revenue and the costs. $P = R - C$

MARGINAL REVENUE: The *rate of change* of the Revenue Function. Often denoted as MR or R'. The first derivative of the Revenue Function. "The approximate amount of revenue one should expect to receive with the sales of ONE more unit."

MARGINAL COST: The rate of change of the Cost Function. Often denoted as MC or C'. "The approximate costs one should expect to pay with the sale of ONE more unit."

MARGINAL PROFIT: The rate of change of the Profit Function. Often denoted as MP or P'. "The approximate profit one would expect when ONE more unit is sold.

Note: The word "*MARGINAL*" can be translated intothe phrase "The first derivative of...". Since we usually interpret "rate of change of" as the derivative of , this is not a new concept.

Economists say, "A firm's most profitable output level occurs where marginal costs equal marginal revenue."

Consider: Since the profit function is nothing more than the difference of two other functions, the first derivative of Profit with respect to the number of units sold, x , (ie. $\frac{dP}{dx}$) is equal to the first derivative of Revenue less the first derivative of Costs.

To find a maximum (or minimum) of any function, we would set the first derivative equal to zero and solve for the critical values of x. We then could use the "second derivative test" to determine if we, in fact, had a min or max.

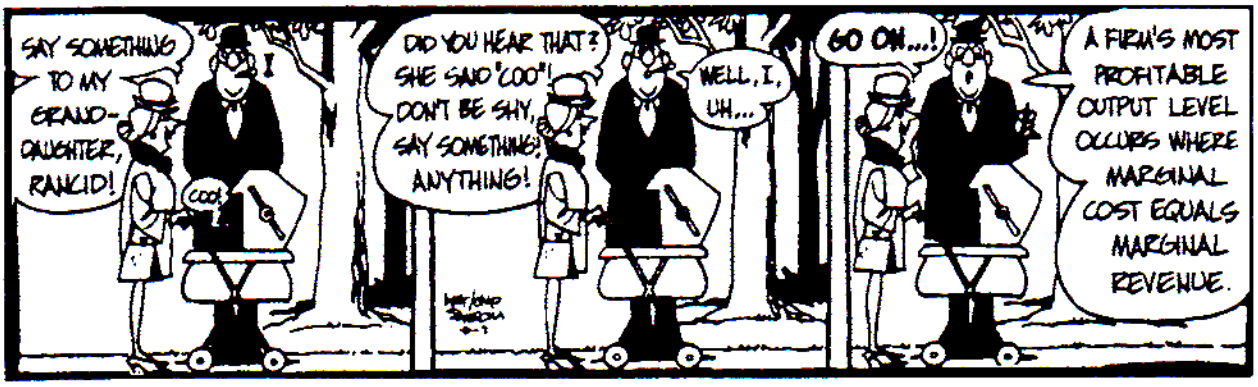
$$P = R - C \qquad \text{Definition}$$

$$P' = R' - C' \stackrel{set}{=} 0 \qquad \text{take derivative; set = 0}$$

$$R' = C' \qquad \text{solve}$$

$$MR = MC \qquad \text{translate to econese}$$

So, the economists are correct. We do have a critical value when the marginal revenue is equal to the marginal costs.


SAY SOMETHING TO MY GRAND-DAUGHTER, RANCID!
coo!
DID YOU HEAR THAT? SHE SAID 'COO'! DON'T BE SHY, SAY SOMETHING! ANYTHING!
WELL, I, UH...
GO ON...!
A FIRM'S MOST PROFITABLE OUTPUT LEVEL OCCURS WHERE MARGINAL COST EQUALS MARGINAL REVENUE.

The scenario: Jake owns a hamburger stand. He had a study done of the people in this area and their desires. The study predicts that He will sell x hamburgers if he charges Q dollars for each hamburger.

The ***Demand Function*** is : $Q = \frac{60000 - x}{20000} = 3 - \frac{x}{20000}$

Jake pays $5,000 a month for his rent and personnel. Each hamburger he sell costs him 56 cents in materials.

the Cost function (C)is : _____

the Revenue function (xQ) is: _____

the Profit function ($R - C$) is: _____

the Marginal Cost function (C') is: _____

This is reasonable since he uses 56 cents worth of materials to make ONE hamburger.

the Marginal Revenue function (R') is: _____

the Marginal Profit function (P') is: _____

Let us consider a few possible situations.:

TABLE # 1

Number of burgers –>	10K	20K	30K	40K	50K
Q = Price/burger	2.50	2.00	1.50	1.00	0.50
R = Total Revenue	25K	40K	45K	40K	25K
C = Total Costs	10.6K	16.2K	21.8K	27.4K	33K
P = Total Profit	14.4K	23.8K	23.2K	12.6K	–8K

1) Notice that from 10K to 20K, he doubled the number of burgers sold but that did not double his profits.
2) Notice that producing 30K burgers will produce less profit than producing 20K burgers. Why does this occur?
3) We expect there to be a point when Jake would achieve maximum profit.

$$P(x) = 2.44x - \frac{x^2}{20000} - 5000$$

$$P'(x) = 2.44 - \frac{x}{10000} \overset{set}{=} 0$$

$$x = 24,400$$

$$P''(x) = -\frac{1}{10000} < 0 \text{ for the critical value so I have a max. } f'' \text{ test.}$$

Fill in **Table # 1** with the following data using x = 24,400 in the above equations to get: Price per burger is $ 1.78 ; total revenue is $ 43,432 ; total costs are $ 18,664 ; and total profit is $ 24,768.

TABLE # 2

Number of burgers –>	10K	20K	30K	40K	50K
MR = R '	2	1	0	–1	–2
MC = C '	.56	.56	.56	.56	.56
MP = P '	1.44	.44	–.56	–1.56	–2.56

Let us look a a slightly different situation. For ease in computation, we shall use the situation when we wish to sell our hamburgers at $ 2.00 each:

From table # 1

We expect to sell 20,000 hamburgers.
We expect to have a gross revenue of $ 40,000.
We expect our expenses to be $ 16,200.
We expect our profit to be $ 23,800.

Look at table # 2.

QUESTION: Do we really expect our marginal revenue to be $ 1.00 when $x = 20{,}000$?

Remember, from our definitions page, Marginal Revenue is the "approximate revenue expected for selling ONE more unit."

Restated: Billy Joe-Bob, Jake's trusted employee, hands out burger number 20,000 of the month.
He collects $ 2.00.
Should Jake expect his revenue to increase by $ 2.00 (the money Billy Joe-Bob collected for the burger) or by $ 1.00 (which the Marginal Revenue function says he should expect)?

The marginal costs is 56 cents. Exactly the cost for making that ONE more burger.

The Marginal Profit is 44 cents, the difference between the $ 1.00 Marginal Revenue and the 56 cents of Marginal Costs.

HOMEWORK:

1) Should Billy Joe-Bob just put the "extra" dollar in his pocket?

2) Does the adage "Approximate amount of revenue one should expect to receive with the sales of ONE more unit." fail?

3) Mathematically, everything looks proper. Is the problem here in the concept of the selling price?

4) Resolve the dilemma.

Our investigation might have us fill in Table #2 with the figures for 24,400 burgers sold.

$R' = .56$, $C' = .56$, and $P' = 0$.

Consider the number of burgers to be 19,999.

	19,999	20,000
Q	2.00005	2.00
R	39,999	40,000
C	16199.44	16200.00
P	23799.56	23800.00
R′	1.0001	1.00
C′	0.56	0.56
P′	0.4401	0.44

My revenue DID increase only one dollar! My "approximate revenue for one more unit" is correct !

Several economics textbooks point out that marginal revenue should be equal to price per unit. However, each of their examples has a demand which is constant !

The answer lies in the fact that average revenue DOES NOT necessarily equal marginal revenue AND that we do NOT have a varying demand !

I must reduce my price per burger to sell more burgers. By assuming that my demand function is NOT constant I am assuming that burger number 20,000 sold for $2.00 AND that burger number 19,999 sold for $ 2.00005 and burger number 19,998 sold for $ 2.00010 etc.

When I decide to sell 20,000 burgers, the demand function says I can sell them at $ 2.00 each. That is, once I have decided upon a sales price, my demand function can be disregarded and NOW I have a horizontal, ie. constant, demand function !

Every burger, ie #1, #123, #1,234, #12,345 and #20,000 will sell at $ 2.00 ! My average revenue per burger does equal my selling price !

The actual use of the demand function is three-fold:

1) It allows speculation about the market size. Should I have an operation capable of 10,000 units, 20,000 units or more?

2) It tells me what price I should use to stimulate a certain demand.

3) It allows me to write my revenue function.

Once these three things have taken place, the demand function ceases to have significance.

We expect our marginal revenue to slow when we are very close to our maximum profit. Remember, Marginal Revenue is really nothing more than the slope of the revenue curve. In our example, that revenue curve is a concave down parabola. It has a maximum. When we near that maximum, the vertical change should be very slight.

Finally, when Joe sold burger number 20,000, the cash register had $ 39,998. I expect it to have $ 40,000 after the 20,000th burger has been sold. Joe should NOT put that "extra" dollar in his pocket.

The key to understanding the apparent mess we're in is realizing that we have a fixed price of $2.00. In other words, $Q = 2$. Now, we have to remember that our revenue function is the price per unit times the number of units sold. Our price is fixed at $2.00 so we do Not have to use the demand fctn for Q because we already have Q's value. Our revenue fctn $= Q_x$ (ie $2x$). That's the difference that caused our previous problem; we were working with the demand fctn, $Q = 3 - \frac{x}{2000}$, to find the revenue fctn instead of our known value of Q, ie 2. By using $Q = 2$ and $R = 2x$, our marginal revenue R' is 2 and Not 1!

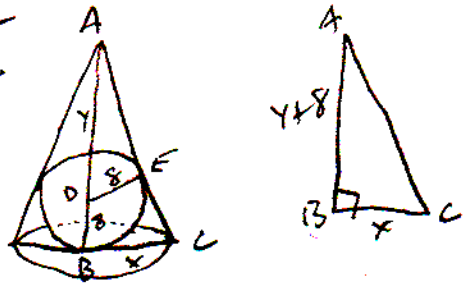
$$Q = 3 - \frac{x}{2000} = 3 - 1 = 2$$ (fixed Now — No further changes allowed)

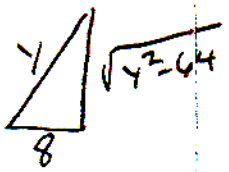
$R = Q_x = 2x$

$R' = 2$

The "approximate revenue from selling one more unit" wasn't wrong. We just failed to account for the fact that Q was no longer a variable. The problem was in the concept of the selling price!

I





$$V = \frac{64\pi}{3}\,\frac{(Y+8)^2}{Y-8}\,4$$

$$V' = \frac{64\pi}{3}\left[\frac{(Y-8)2(Y+8)-(Y+8)^2}{(Y-8)^2}\right]$$

$$= \frac{64\pi}{3}\,\frac{(Y+8)(2Y-16-Y-8)}{(Y-8)^2}$$

$$V' = \frac{64\pi}{3}\,\frac{(Y+8)(Y-24)}{(Y-8)^2} \overset{\text{set}}{=} 0$$

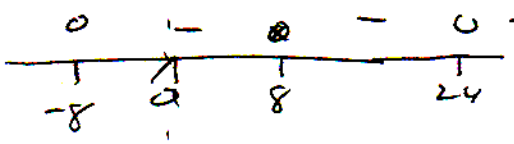
$$\frac{x}{8} = \frac{Y+8}{\sqrt{Y^2-64}}$$

$$x = \frac{8(Y+8)}{\sqrt{Y^2-64}}$$

$V = \frac{1}{3}\pi x^2 4$

$V = \frac{\pi}{3}\,\frac{64(Y+8)^2}{Y^2-64}\cdot Y+8$

$Y-8$



$Y>0$ ∴ min at $Y=24$ f'test.

$$x = \frac{8(32)}{\sqrt{24^2-64}} = \frac{8(32)}{\sqrt{8\cdot8\cdot3\cdot3+8\cdot8}}$$

$$x = \frac{8\cdot8\cdot4}{8\sqrt{9-1}} = \frac{16\sqrt{2}}{\sqrt{2}\sqrt{2}} = 8\sqrt{2}$$

radius $8\sqrt{2}$ height 32

II

$$C(x) = \frac{x}{4} + 35 + \frac{25}{x}$$

Price $Q = 50 - \frac{x}{2}$

$$R = 50x - \frac{x^2}{2} - \left(\frac{x^2}{4} + 35x + 25\right)$$

$$= 50x - \frac{3x^2}{4} - 35x - 25$$

$$R = 15x - \frac{3x^2}{4} - 25$$

$$R' = 15 - \frac{3x}{2} \overset{\text{set}}{=} 0$$

$$\frac{3}{2}x = 15\ \frac{2}{3}$$

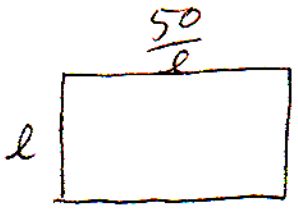
$$x = 10$$

$R'' = -\frac{3}{2} < 0$ ∴

max f'test.

They need to sell 10 radios per day.

III



$\frac{d\ell}{dt} = 2$

$\frac{dP}{dt} = ?$

$P = 2\ell + \frac{100}{\ell}$

$\frac{dP}{dt} = 2\frac{d\ell}{dt} - \frac{100}{\ell^2}\frac{d\ell}{dt}$

c)

$2\frac{d\ell}{dt} - \frac{100}{\ell^2}\frac{d\ell}{dt} = 0$

$4 - \frac{200}{\ell^2} = 0$

$4\ell^2 = 200$

$\ell^2 = 50$

$\ell = 5\sqrt{2}$

perimeter stops changing when $\frac{dP}{dt} = 0$ when rectangle is a square $5\sqrt{2}$ on a side

4) $\frac{dP}{dt} = 2(2) - \frac{100}{25}(2)$

$= 4 - 8$

$\frac{dP}{dt} = -4$ in

perimeter shortens at 4"/sec

B) $\frac{dP}{dt} = 2(2) - \frac{100}{100}(2)$

$= 4 - 2$

$\frac{dP}{dt} = 2$ perimeter grows at 2"/sec.

IV

$s = 64t - 16t^2$

$48 = 64t - 16t^2$

$16t^2 - 64t + 48$

$16(t^2 - 4t + 3)$

$16(t-3)(t-1)$

at $t = 1$ $s = 48$

when $t = 3$ $s = 48$ on downward flight.

$v = 64 - 32t$

$v(0) = 64$

$v(1) = 32$

clearly $v(1)$ is $\frac{1}{2}$ of $v(0)$ the initial velocity.

90 Liz Silow 62
93 Becky Mercer 65
97 Helen Romelashvili 98
98 Andrew Hahn 75
Hoin Park 60
Ravin Singh 20
2000 Dustin Dolcourt 40

2003 Lia Wyp 40
Tiffany Hoff 30
Daniel Hitzeman 10

} also failed, Runion Matt Lia was not a Sumpter.

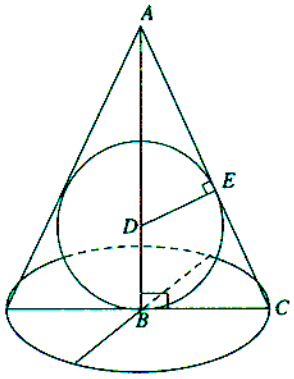
Honors Analysis Name AnLCh 4A

Copy original problem. Per _____ Date Make up

Convince *me* that **you** understand the concept*!*

Chapter 4 Applications Make Up Exam

I A right circular cone is circumscribed about a sphere of radius 8 inches. See figure. $\overline{DE} = 8 = \overline{BD}$; $\measuredangle DEA = \measuredangle DBC = 90^\circ$. (hint: let $\overline{AD} = y$ and $\overline{BC} = x$. $\Delta AED \approx \Delta ABC$ both right triangles have a common angle.)



$V_{cone} = \frac{1}{3}\pi r^2 h$ (25 pts tot)

a) Use the hints given above and show that the volume can be represented by

$$V = \frac{64\pi}{3}\,\frac{(y+8)^2}{y-8}$$

b) Find the dimensions of the cone which has minimum volume.

II The cost of producing a single radio in dollars is: $\frac{x}{4} + 35 + \frac{25}{x}$.

The price per set at which they may be sold is $50 - \frac{x}{2}$.

What should be the daily output to obtain maximum profit? (25 pts)

III Two parallel sides of a rectangle are being lengthened at the rate of two inches per second while the other two sides are shortened in such a way that the figure remains a rectangle with constant area of fifty square inches. What is the rate of change of the perimeter when the length of an increasing side is ...

(tot 35 pts)

a) 5 inches

b) 10 inches

c) What are the dimensions when the perimeter ceases to decrease?

IV A body moves vertically up from the Earth according to : $s = 64t - 16t^2$.

s is the distance above the launch height. v, the first derivative of s WRT t, is the velocity. Show that the body has lost one-half of its velocity in the first forty-eight feet of rise. (15 pts)